# Abstract Linear Algebra - Part 3

$\boxed{\text{set + 8 rules}}$ — $V$ $\mathbb{F}$-vector space

↳ for example: space of functions

↳ zero vector $o \in V$

**Question:** $\underset{\uparrow \text{ zero in } \mathbb{F}}{0} \cdot v = o$ ← zero vector , $(-1)\cdot v = -v$ for $v \in V$ ?

**Proof:** $0 \cdot v = (0+0)\cdot v \overset{(8)}{=} 0\cdot v + 0 \cdot v$ ← associativity (1)

$$\overset{(3)}{\Longrightarrow} \quad 0\cdot v + \big(-(0\cdot v)\big) = 0 \cdot v + \underbrace{\Big(0\cdot v + \big(-(0\cdot v)\big)\Big)}_{=\,o}$$

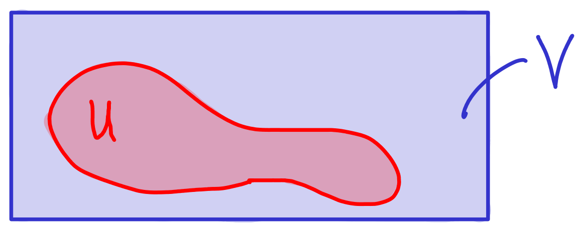
$$\overset{(3)}{\Longrightarrow} \quad o = 0 \cdot v \quad \checkmark$$

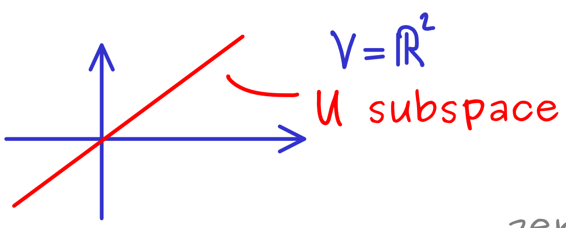
$$= \big(1 + (-1)\big)\cdot v \overset{(8)}{=} \underbrace{1\cdot v}_{(6)\nearrow\, v} + (-1)\cdot v$$

$$\overset{(3)}{\Longrightarrow} \quad -v + o = \underbrace{-v + v}_{=\,o} + (-1)\cdot v \quad \Longrightarrow \quad -v = (-1)\cdot v \quad \checkmark$$

**Linear subspace:**

- vector space inside another one




- 


- $\mathcal{P}(\mathbb{R}) \subseteq \mathcal{F}(\mathbb{R})$
  - zero function lies in $\mathcal{P}(\mathbb{R})$
  - adding two polynomials gives polynomial
  - scaling polynomial gives polynomial

**Definition:** $V$ $\mathbb{F}$-vector space, $U \subseteq V$. If

(a) $o \in U$,

(b) $u, v \in U \implies u + v \in U$,

(c) $u \in U, \lambda \in \mathbb{F} \implies \lambda \cdot u \in U$,

then $U$ is also an $\mathbb{F}$-vector space. We call it a linear subspace of $V$.

**Example:** $\mathcal{P}_2(\mathbb{R})$ polynomials with degree $\leq 2$ $\quad (x \mapsto 4x^2 + x, \; x \mapsto 8x + 1)$

$$\implies \mathcal{P}_2(\mathbb{R}) \subseteq \mathcal{F}(\mathbb{R}) \quad \text{subspace}$$