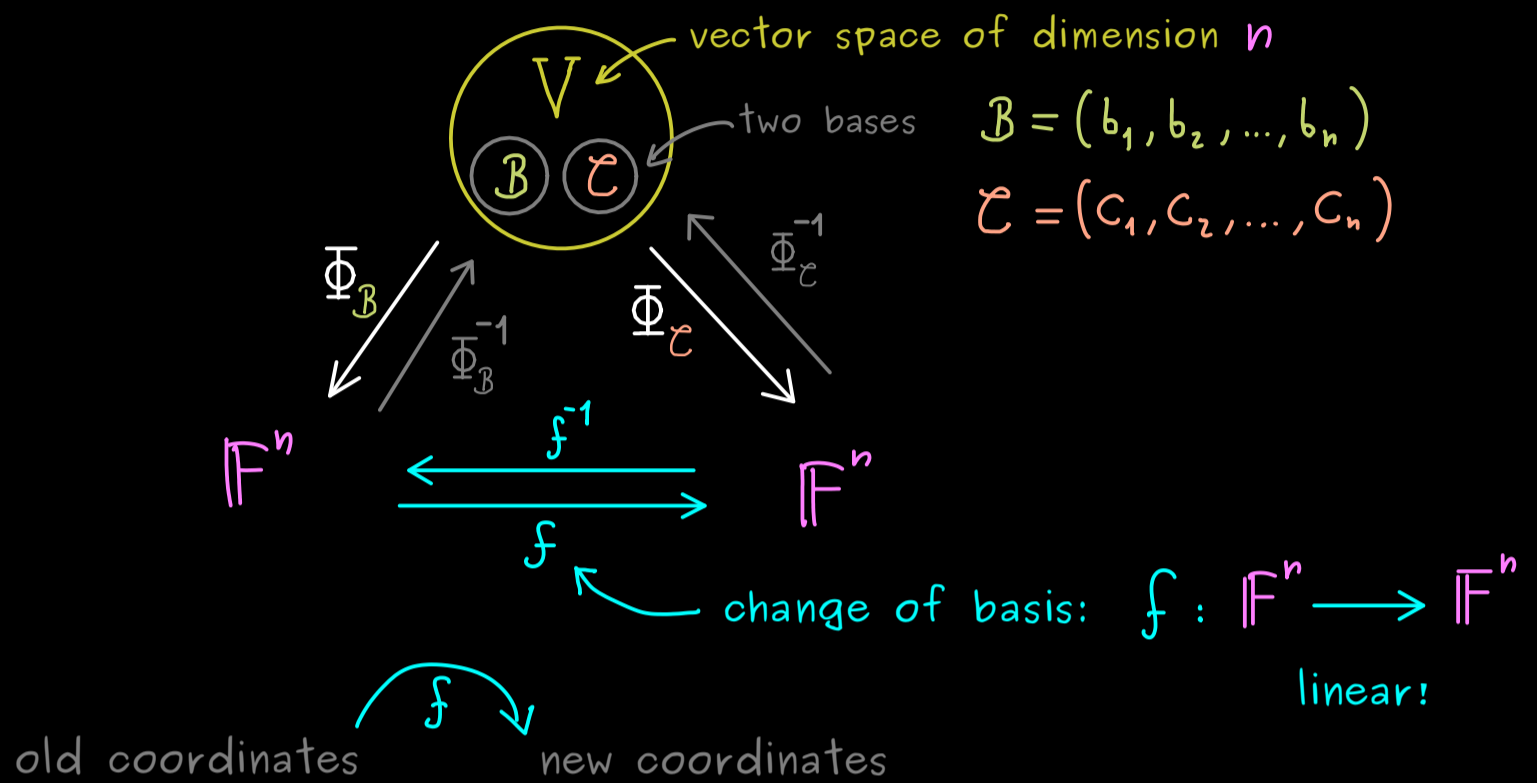




## Abstract Linear Algebra - Part 8

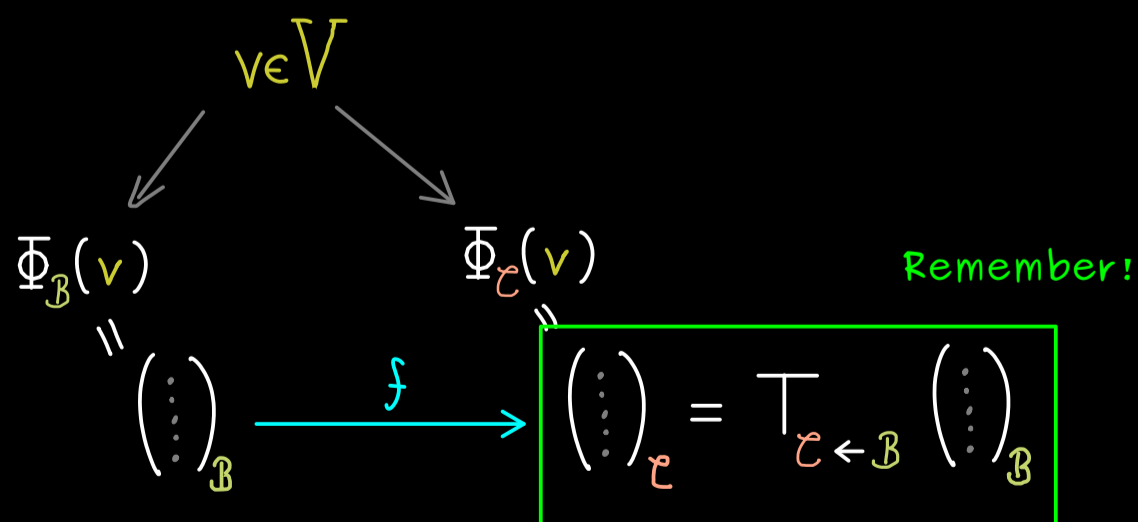


What happens if we put  $e_1 = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$  into  $f$ ?  $\leadsto f(e_1) = \Phi_{\mathcal{C}} \left( \underbrace{\Phi_{\mathcal{B}}^{-1}(e_1)}_{b_1} \right) = \Phi_{\mathcal{C}}(b_1)$

We can see  $f$  as a matrix  $f(x) = \begin{pmatrix} \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \end{pmatrix} \begin{pmatrix} | \\ | \\ | \\ | \end{pmatrix} x$

$$T_{\mathcal{C} \leftarrow \mathcal{B}} = \begin{pmatrix} | & | & & | \\ \Phi_{\mathcal{C}}(b_1) & \Phi_{\mathcal{C}}(b_2) & \dots & \Phi_{\mathcal{C}}(b_n) \\ | & | & & | \end{pmatrix}$$

transformation matrix  
 transition matrix  
 change-of-basis matrix  
 from  $\mathcal{B}$  to  $\mathcal{C}$



Fact:  $\left( T_{\mathcal{C} \leftarrow \mathcal{B}} \right)^{-1} = T_{\mathcal{B} \leftarrow \mathcal{C}}$

Example:  $V = \mathcal{P}_2(\mathbb{R})$  polynomials of degree  $\leq 2$

$$m_0: X \mapsto 1$$

$$m_1: X \mapsto X$$

$$m_2: X \mapsto X^2$$

$$\mathcal{B} = (\underbrace{m_2}_{b_1}, \underbrace{m_1}_{b_2}, \underbrace{m_0}_{b_3})$$

$$\mathcal{C} = (\underbrace{m_2 - \frac{1}{2}m_1}_{c_1}, \underbrace{m_2 + \frac{1}{2}m_1}_{c_2}, \underbrace{m_0}_{c_3})$$

$T_{\mathcal{C} \leftarrow \mathcal{B}}$   $\rightsquigarrow$  how to write  $b_j$  with a linear combination of  $\mathcal{C}$

$T_{\mathcal{B} \leftarrow \mathcal{C}}$   $\rightsquigarrow$  how to write  $c_j$  with a linear combination of  $\mathcal{B}$

$\hookrightarrow$  column vectors  $\Phi_{\mathcal{B}}(c_1) = \begin{pmatrix} 1 \\ -\frac{1}{2} \\ 0 \end{pmatrix}$ ,  $\Phi_{\mathcal{B}}(c_2) = \begin{pmatrix} 1 \\ \frac{1}{2} \\ 0 \end{pmatrix}$ ,  $\Phi_{\mathcal{B}}(c_3) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$$T_{\mathcal{B} \leftarrow \mathcal{C}} = \begin{pmatrix} 1 & 1 & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow{\text{calculate inverse!}} T_{\mathcal{C} \leftarrow \mathcal{B}}$$