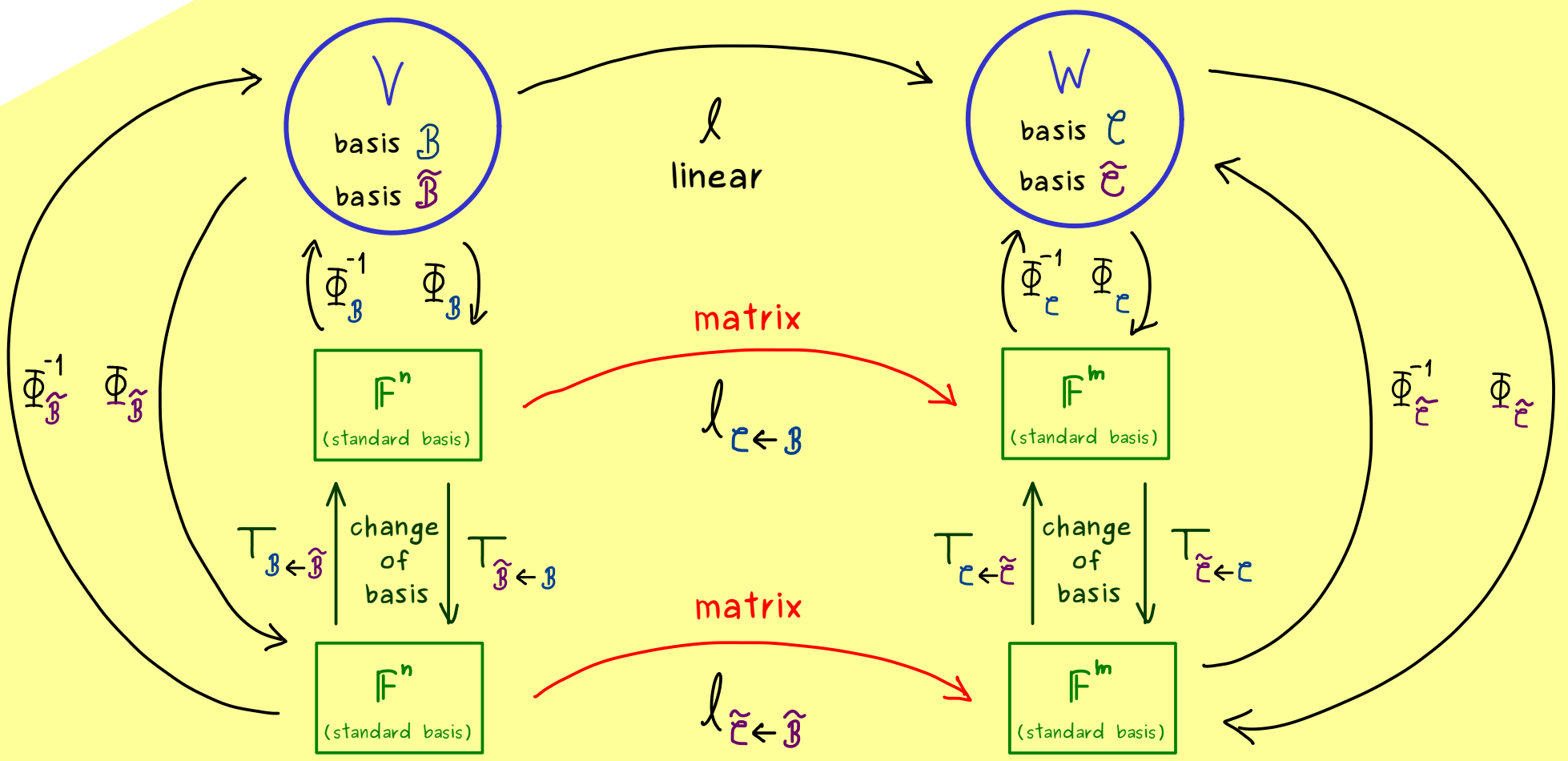




Abstract Linear Algebra - Part 27



Result:

$$l_{\tilde{C} \leftarrow \tilde{B}} = T_{\tilde{C} \leftarrow C} l_{C \leftarrow B} T_{B \leftarrow \tilde{B}}$$

Example: $l: \mathcal{P}_3(\mathbb{R}) \rightarrow \mathcal{P}_2(\mathbb{R})$, $l(p) = p'$ linear map!

$$B = (m_3, m_2, m_1, m_0) \quad C = (m_2, m_1, m_0)$$

$$\tilde{B} = (2m_3 - m_1, m_2 + m_0, m_1 + m_0, m_1 - m_0), \quad \tilde{C} = (m_2 - \frac{1}{2}m_1, m_2 + \frac{1}{2}m_1, m_0)$$

matrix representation: $l_{C \leftarrow B} = \begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

change-of-basis matrices: $T_{B \leftarrow \tilde{B}} = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{pmatrix}$

$$T_{C \leftarrow \tilde{C}} = \begin{pmatrix} 1 & 1 & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow{\text{inverse}} T_{\tilde{C} \leftarrow C} = \begin{pmatrix} \frac{1}{2} & -1 & 0 \\ \frac{1}{2} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{aligned} l_{\tilde{C} \leftarrow \tilde{B}} &= T_{\tilde{C} \leftarrow C} l_{C \leftarrow B} T_{B \leftarrow \tilde{B}} = \begin{pmatrix} \frac{1}{2} & -1 & 0 \\ \frac{1}{2} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{pmatrix} \\ &= \begin{pmatrix} \frac{1}{2} & -1 & 0 \\ \frac{1}{2} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 6 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ -1 & 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 3 & -2 & 0 & 0 \\ 3 & 2 & 0 & 0 \\ -1 & 0 & 1 & 1 \end{pmatrix} \end{aligned}$$