



## Abstract Linear Algebra - Part 28

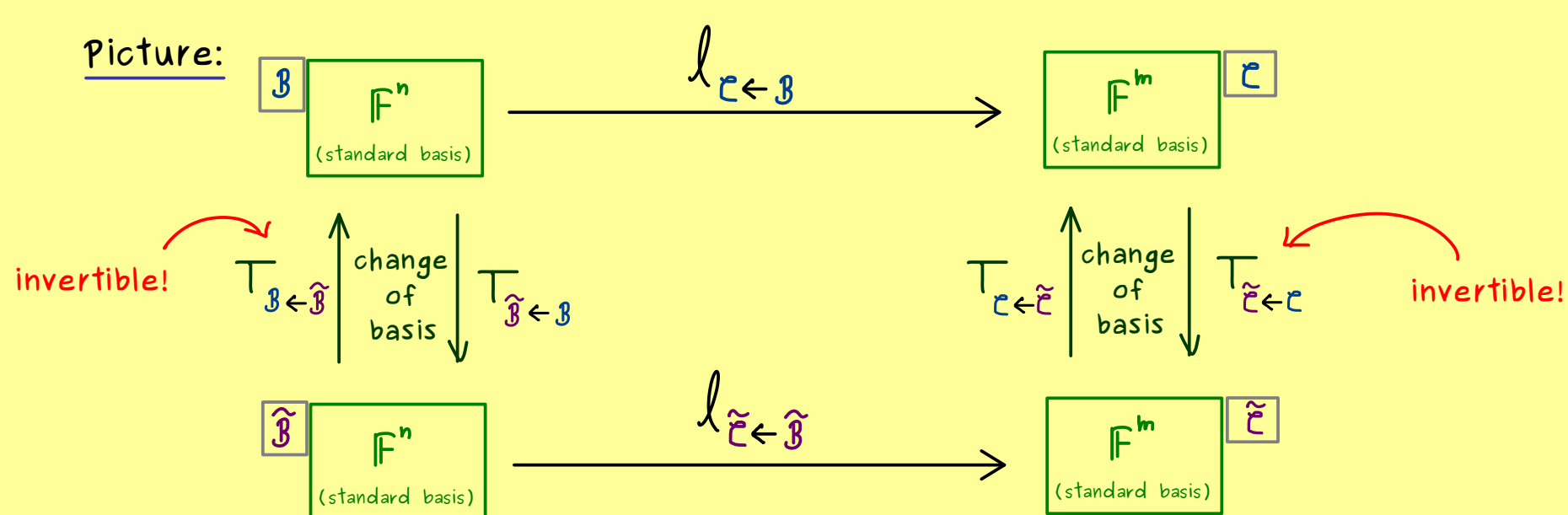
Fact:  $\begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$  and  $\begin{pmatrix} 3 & -2 & 0 & 0 \\ 3 & 2 & 0 & 0 \\ -1 & 0 & 1 & 1 \end{pmatrix}$  are different but

they describe the same linear map  $\ell: \mathcal{P}_3(\mathbb{R}) \rightarrow \mathcal{P}_2(\mathbb{R})$ ,  $\ell(p) = p'$  with respect to different bases.

Question:  $\ell: V \rightarrow W$  linear,  $A = \ell_{\mathcal{C} \leftarrow \mathcal{B}} \in \mathbb{F}^{m \times n}$ .

For another  $\tilde{A} \in \mathbb{F}^{m \times n}$ , can we find bases such that  $\tilde{A} = \ell_{\tilde{\mathcal{C}} \leftarrow \tilde{\mathcal{B}}}$ ?

If **YES!**, then we say  $A$  and  $\tilde{A}$  are equivalent.



Definition: A matrix  $\tilde{A} \in \mathbb{F}^{m \times n}$  is called equivalent to a matrix  $A \in \mathbb{F}^{m \times n}$

if there are invertible matrices  $S \in \mathbb{F}^{m \times m}$ ,  $T \in \mathbb{F}^{n \times n}$ , such that:

$$\tilde{A} = S A T.$$

We write:  $\tilde{A} \sim A$

Remark:  $\sim$  defines an equivalence relation on  $\mathbb{F}^{m \times n}$ :

(1) reflexive:  $A \sim A$  for all  $A \in \mathbb{F}^{m \times n}$

(2) symmetric:  $A \sim B \Rightarrow B \sim A$  for all  $A, B \in \mathbb{F}^{m \times n}$

(3) transitive:  $A \sim B \wedge B \sim C \Rightarrow A \sim C$  for all  $A, B, C \in \mathbb{F}^{m \times n}$