ON STEADY

The Bright Side of Mathematics



Algebra - Part 9



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<u>Definition:</u>  $(G, \circ), (H, *)$  groups. A map  $\varphi: G \longrightarrow H$  is called a group homomorphism if  $\varphi(a \circ b) = \varphi(a) * \varphi(b)$  for all  $a, b \in G$ .

Example: 
$$(G, \circ) = (\mathbb{R}, +), (H, *) = (\mathbb{R} \setminus \{0\}, \cdot).$$
  
 $\psi: G \longrightarrow H$   
 $x \mapsto e^{x} \implies \psi(x + y) = e^{x + y}, \psi(x) \cdot \psi(y) = e^{x} \cdot e^{y}$ 

Properties: A group homomorphism satisfies:

(1) 
$$\psi(e_G) = e_H$$
 (identity is sent to identity)  
(2)  $\psi(a^1) = \psi(a)^{-1}$  for all  $a \in G$ .

$$\frac{\text{Proof:}}{\Rightarrow} (1) \quad \psi(e_G) = \psi(e_G \circ e_G) = \psi(e_G) * \psi(e_G)$$

$$\Rightarrow e_H = \psi(e_G)^{-1} * \psi(e_G) = \psi(e_G)^{-1} * (\psi(e_G) * \psi(e_G))$$

$$= (\psi(e_G)^{-1} * \psi(e_G)) * \psi(e_G) = \psi(e_G)$$

$$= e_H$$

$$^{(2)} e_{H} = \varphi(e_{G}) = \varphi(\bar{a}^{1} \cdot a) = \varphi(\bar{a}^{-1}) * \varphi(a)$$

inverse unique 
$$-1 = \varphi(a^{-1})$$