Introduction and Open Sets in Metric Spaces

Basic Topology - Part 1

Real Analysis → Basic Topology
Multivariable Calculus → Basic Topology

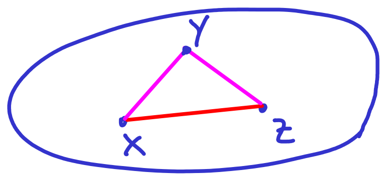
Basic Topology ⟷ Manifolds
Basic Topology ⟷ Functional Analysis
Basic Topology → Algebraic Topology

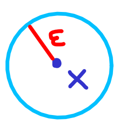
Basic Topology →
- topological spaces
- Hausdorff spaces
- locally compact spaces
- ⋮

Metric spaces: A set X together with a metric $d : X \times X \longrightarrow [0,\infty)$ is called a metric space. (written (X,d))

- positive definite: $d(x,y) = 0 \Leftrightarrow x = y$
- symmetric: $d(x,y) = d(y,x)$
- triangle inequality: $d(x,z) \leq d(x,y) + d(y,z)$

$$B_\varepsilon(x) := \{ y \in X \mid d(x,y) < \varepsilon \}$$



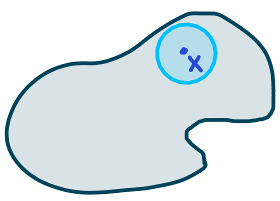


open ball of radius ε centered at x

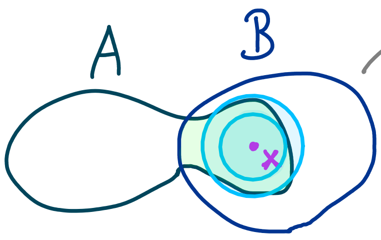
$\varepsilon \searrow 0$

Open sets in a metric space: (X,d) metric space, $A \subseteq X$.

A is called open if for all $x \in X$ there is $\varepsilon > 0$ such that $B_\varepsilon(x) \subseteq A$.



Properties:
(1) $\emptyset, X$ are open



(2) A, B open $\Longrightarrow$ $A \cap B$ open

(3) A_i open for all $i \in I$ $\Longrightarrow$ $\bigcup_{i \in I} A_i$ open