



Calculating dimension and basis of range and kernel

($\leadsto$ row echelon form)

$$A = \begin{pmatrix} 2 & 1 & 3 & 2 \\ 4 & 2 & 1 & 1 \\ 8 & 4 & 17 & 11 \end{pmatrix} \xrightarrow[\text{III} - 4\text{I}]{\text{II} - 2\text{I}} \begin{pmatrix} 2 & 1 & 3 & 2 \\ 0 & 0 & -5 & -3 \\ 0 & 0 & 5 & 3 \end{pmatrix}$$

$$\xrightarrow{\text{III} + \text{II}} \begin{pmatrix} 2 & 1 & 3 & 2 \\ 0 & 0 & -5 & -3 \\ 0 & 0 & 0 & 0 \end{pmatrix} = A'$$

$\uparrow$ free variables: x_2, x_4

Read dimensions from
the row echelon form:

$$\dim(\text{Ker}(A)) = 2$$

$$\dim(\text{Ran}(A)) = 2$$

$$\text{Only row operations} \Rightarrow \text{Ker}(A) = \text{Ker}(A') \\ \text{Ran}(A) \neq \text{Ran}(A') \quad (\text{in general})$$

$$\text{For calculating Ker}(A): \begin{cases} 2x_1 + 1x_2 + 3x_3 + 2x_4 = 0 \\ -5x_3 - 3x_4 = 0 \end{cases} \Rightarrow \begin{cases} 2x_1 = -x_2 - 3x_3 - 2x_4 \\ x_3 = -\frac{3}{5}x_4 \end{cases}$$

$$\text{Ker}(A) = \left\{ \alpha \cdot \begin{pmatrix} -1 \\ 2 \\ 0 \\ 0 \end{pmatrix} + \beta \cdot \begin{pmatrix} -1 \\ 0 \\ -6 \\ 10 \end{pmatrix} \mid \alpha, \beta \in \mathbb{R} \right\}$$

$$\text{A basis for Ker}(A) \text{ is } \left(\begin{pmatrix} -1 \\ 2 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ -6 \\ 10 \end{pmatrix} \right).$$

$$\text{A basis for Ran}(A) \text{ is } \left(\begin{pmatrix} 2 \\ 4 \\ 8 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ 17 \end{pmatrix} \right).$$