

# Calculating dimension and basis of range and kernel

( $\rightarrow$  row echelon form)

$$A = \begin{pmatrix} 2 & 1 & 3 & 2 \\ 4 & 2 & 1 & 1 \\ 8 & 4 & 17 & 11 \end{pmatrix} \begin{array}{l} \text{II} - 2\text{I} \\ \text{III} - 4\text{I} \end{array} \rightsquigarrow \begin{pmatrix} 2 & 1 & 3 & 2 \\ 0 & 0 & -5 & -3 \\ 0 & 0 & 5 & 3 \end{pmatrix}$$

$$\begin{array}{l} \text{III}' + \text{II}' \\ \rightsquigarrow \end{array} \begin{pmatrix} 2 & 1 & 3 & 2 \\ 0 & 0 & -5 & -3 \\ 0 & 0 & 0 & 0 \end{pmatrix} = A'$$

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 free variables:  $x_2, x_4$

Read dimensions from the row echelon form:

$$\dim(\text{Ker}(A)) = 2$$

$$\dim(\text{Ran}(A)) = 2$$

$$\text{Only row operations} \Rightarrow \text{Ker}(A) = \text{Ker}(A')$$

$$\text{Ran}(A) \neq \text{Ran}(A') \quad (\text{in general})$$

For calculating  $\text{Ker}(A)$ :  $2x_1 + 1x_2 + 3x_3 + 2x_4 = 0$   
 $-5x_3 - 3x_4 = 0$

$$\Rightarrow \begin{array}{l} 2x_1 = -x_2 - 3x_3 - 2x_4 \\ x_3 = -\frac{3}{5}x_4 \end{array}$$

$$\text{Ker}(A) = \left\{ \alpha \cdot \begin{pmatrix} -1 \\ 2 \\ 0 \\ 0 \end{pmatrix} + \beta \cdot \begin{pmatrix} -1 \\ 0 \\ -6 \\ 10 \end{pmatrix} \mid \alpha, \beta \in \mathbb{R} \right\}$$

A basis for  $\text{Ker}(A)$  is  $\left( \begin{pmatrix} -1 \\ 2 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ -6 \\ 10 \end{pmatrix} \right)$ .

A basis for  $\text{Ran}(A)$  is  $\left( \begin{pmatrix} 2 \\ 4 \\ 8 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ 17 \end{pmatrix} \right)$ .