Calculating dimension and basis of range and kernel

(→ row echelon form)

$$A = \begin{pmatrix} 2 & 1 & 3 & 2 \\ 4 & 2 & 1 & 1 \\ 8 & 4 & 17 & 11 \end{pmatrix} \xrightarrow[\text{III}-4\text{I}]{\text{II}-2\text{I}} \begin{pmatrix} 2 & 1 & 3 & 2 \\ 0 & 0 & -5 & -3 \\ 0 & 0 & 5 & 3 \end{pmatrix}$$

$$\xrightarrow{\text{III}' + \text{II}'} \begin{pmatrix} 2 & 1 & 3 & 2 \\ 0 & 0 & -5 & -3 \\ 0 & 0 & 0 & 0 \end{pmatrix} = A'$$

pivot, pivot

free variables: x_2, x_4

Read dimensions from the row echelon form:

$$\dim(\mathrm{Ker}(A)) = 2$$

$$\dim(\mathrm{Ran}(A)) = 2$$

Only row operations $\Rightarrow$ $\mathrm{Ker}(A) = \mathrm{Ker}(A')$

$\mathrm{Ran}(A) \neq \mathrm{Ran}(A')$ (in general)

For calculating $\mathrm{Ker}(A)$:

$$\begin{aligned} 2x_1 + 1x_2 + 3x_3 + 2x_4 &= 0 \\ -5x_3 - 3x_4 &= 0 \end{aligned} \Rightarrow \begin{aligned} 2x_1 &= -x_2 - 3x_3 - 2x_4 \\ x_3 &= -\tfrac{3}{5}x_4 \end{aligned}$$

basis

$$\mathrm{Ker}(A) = \left\{ \alpha \cdot \begin{pmatrix} -1 \\ 2 \\ 0 \\ 0 \end{pmatrix} + \beta \cdot \begin{pmatrix} -1 \\ 0 \\ -6 \\ 10 \end{pmatrix} \;\middle|\; \alpha, \beta \in \mathbb{R} \right\}$$

A basis for $\mathrm{Ker}(A)$ is $\left(\begin{pmatrix} -1 \\ 2 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ -6 \\ 10 \end{pmatrix} \right)$.

A basis for $\mathrm{Ran}(A)$ is $\left(\begin{pmatrix} 2 \\ 4 \\ 8 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ 17 \end{pmatrix} \right)$.