Complex Analysis – Part 1

analysis of differentiable functions $f: \mathbb{C} \to \mathbb{C}$

(instead of $f: \mathbb{R} \to \mathbb{R}$)

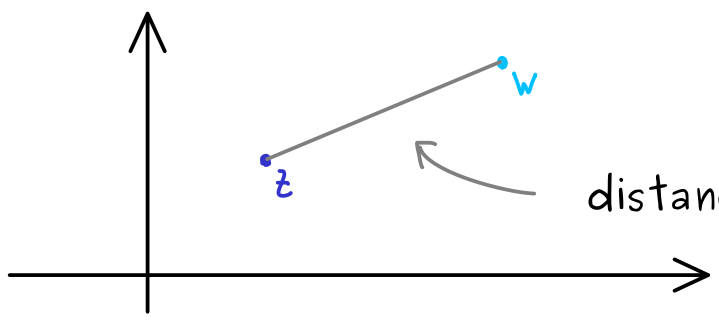
$\mathbb{R} \subseteq \mathbb{C}$ $\Rightarrow$ helpful for real problems like $\displaystyle\int_{-\infty}^{\infty} \frac{x \cdot \sin(x)}{1+x^2}\, dx = \frac{\pi}{e}$

We need:

- sets
- complex numbers } Start Learning Mathematics
- basic knowledge of continuous and differentiable functions
- basic knowledge of power series

Real Analysis (some videos)

Some definitions: $\mathbb{C}$ is a set with a distance (metric space)



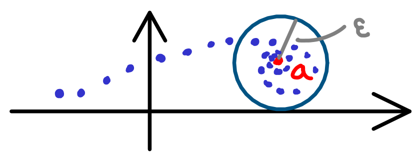
distance: $|z-w|$

absolute value in $\mathbb{C}$

A sequence $(z_n)_{n\in\mathbb{N}} \subseteq \mathbb{C}$ is convergent to $a \in \mathbb{C}$

$\Leftrightarrow$ $(|z_n - a|)_{n\in\mathbb{N}} \subseteq \mathbb{R}$ is convergent to 0

$\Leftrightarrow$ $\forall \varepsilon > 0 \;\; \exists N \in \mathbb{N} \;\; \forall n \geq N : \;\; |z_n - a| < \varepsilon$



ε-ball: $B_\varepsilon(a) := \{ w \in \mathbb{C} \mid |w-a| < \varepsilon \}$

A function $f: \mathbb{C} \to \mathbb{C}$ is continuous at $z_0 \in \mathbb{C}$ if for all sequences $(z_n)_{n\in\mathbb{N}} \subseteq \mathbb{C}$:

$z_n \xrightarrow{n\to\infty} z_0$ implies $f(z_n) \xrightarrow{n\to\infty} f(z_0)$.

means: $(z_n)_{n\in\mathbb{N}}$ is convergent to z_0