Distributions - Part 1

Analysis: Function, limit, derivative

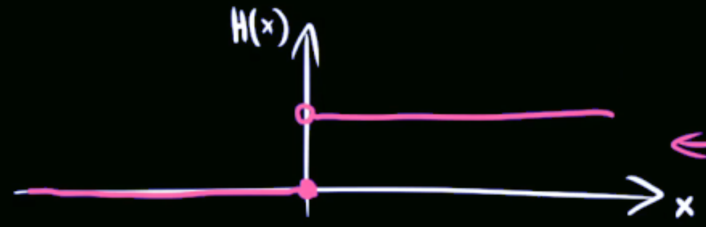
Differential equations, Fourier series, Fourier transform

→ Solutions with sharp turns

→ more general solutions

Historical example: 1927 Paul Dirac, $H: \mathbb{R} \to \mathbb{R}$

Heaviside function



← Derivative?

- classical derivative has a problem at $x = 0$
- "more general" derivative should be "delta function" δ!

$\delta(x) = 0$ for $x \neq 0$, and

for $\varepsilon > 0$: $\int_{-\varepsilon}^{\varepsilon} \delta(x)\, dx = \int_{-\varepsilon}^{\varepsilon} H'(x)\, dx = H(\varepsilon) - H(-\varepsilon) = 1 - 0 = 1$

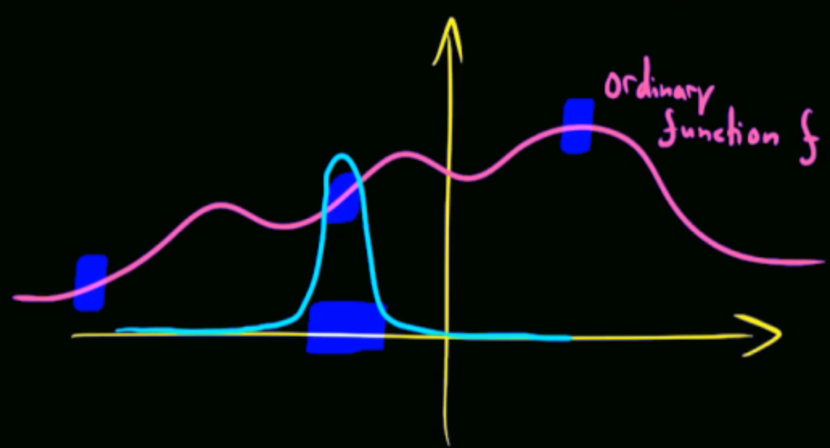
- δ can't be an ordinary function

$\delta(x) = 0$ almost everywhere (w.r.t. Lebesgue measure)

hence $\int_{-\varepsilon}^{\varepsilon} \delta(x) = 0 \neq 1$

- Dirac wants to calculate with $\delta', \delta'', \delta''', \ldots$ → meaning?

$\Rightarrow$ δ will be defined as a distribution → "generalised function"



Use so-called test functions $\varphi: \mathbb{R} \to \mathbb{R}$

$\varphi \mapsto \int_{\mathbb{R}} f(x)\, \varphi(x)\, dx \in \mathbb{R}$ ← linear map

Delta function:

$\varphi \mapsto \varphi(0)$