

Distributions - Part 1

Motivation and Delta Function

Analysis: Function, limit, derivative

Differential equations, Fourier series, Fourier transform
↳ solutions with sharp turns
↳ more general solutions

Historical example:



- classical derivative has a problem at $x=0$
- "more general" derivative should be "delta function" δ !

for $\epsilon > 0$: $\int_{-\epsilon}^{\epsilon} \delta(x) dx = \int_{-\epsilon}^{\epsilon} H'(x) dx = H(\epsilon) - H(-\epsilon) = 1 - 0 = 1$

$\delta(x) = 0$ for $x \neq 0$, and

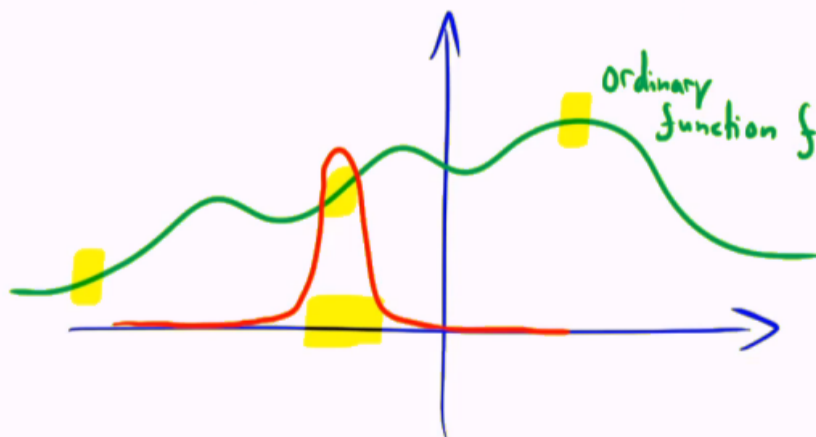
- δ can't be an ordinary function

$\delta(x) = 0$ almost everywhere (v.r.t. Lebesgue measure)

hence $\int_{-\epsilon}^{\epsilon} \delta(x) dx = 0 \neq 1$

- Dirac wants to calculate with $\delta', \delta'', \delta''' \dots$ → meaning?

$\Rightarrow \delta$ will be defined as a distribution → "generalised function"



Use so-called test functions
 $\varphi: \mathbb{R} \rightarrow \mathbb{R}$

$\varphi \mapsto \int_{\mathbb{R}} \delta(x) \varphi(x) dx \in \mathbb{R}$

linear map
 $\varphi \mapsto \varphi(0)$

Delta function:

