



# The Bright Side of Mathematics

## Determinants and Complex Numbers

Exercise 2. Find the complex number  $z \in \mathbb{C}$  that solves the following equation:

$$\left( \frac{1-2j}{2-j} - \frac{3+2j}{1+2j} \right) \cdot \bar{z} = \frac{j-1}{j}$$

$$\bar{z} = \text{conjugate of } z$$

$$z = a + bj \quad \begin{array}{l} \text{Re}(z) = a \\ \text{Im}(z) = b \end{array}$$

$$\bar{z} = a - bj$$

$$\frac{b}{a} \left| \frac{1+2j}{2} \right|$$

$$z \cdot \bar{z} = (a+bj)(a-bj)$$

$$= a^2 - \cancel{baj} + \cancel{abj} - b^2j^2$$

$$= a^2 + b^2 = |z|^2$$

$$\frac{1-2j}{(2-j)(2+j)} \cdot \frac{2+j-4j-2j^2}{4+1} = \frac{2-3j+2}{5} = \frac{4-3j}{5}$$

$$\frac{3+2j}{(1+2j)(1-2j)} = \frac{3-6j+2j-4j^2}{1+4} = \frac{3-4j+4}{5} = \frac{7-4j}{5}$$

$$\frac{4-3j-7+4j}{5} \cdot \bar{z} = \frac{j-1}{j}$$

$$\frac{-3+j}{5} \bar{z} = \frac{j-1}{j}$$

$$\bar{z} = \frac{(j-1)5}{j(-3+j)} = \frac{5(j-1)(3j-1)}{(-3j-1)(3j-1)} = \frac{5(3j^2-j-3j+1)}{10} = \frac{5(-2-4j)}{10}$$

$$\bar{z} = -1-2j \quad \Rightarrow \quad z = -1+2j$$