Determinants and Complex Numbers

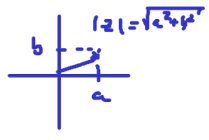
Exercise 2. Find the complex number $z \in \mathbb{C}$ that solves the following equation:

$$\left(\frac{1-2j}{2-j} - \frac{3+2j}{1+2j} \right) \cdot \bar{z} = \frac{j-1}{j}$$

$\bar{z}$ conjugate of z

$z = a + bj$ $\quad Re(z) = a$, $\quad Im(z) = b$

$\bar{z} = a - bj$



$$z \cdot \bar{z} = (a+bj)(a-bj)$$
$$= a^2 - baj + abj - b^2 j^2$$
$$= a^2 + b^2 = |z|^2$$

$$\frac{1-2j}{(2-j)} \frac{(2+j)}{(2+j)} = \frac{2+j-4j-2j^2}{4+1} = \frac{2-3j+2}{5} = \frac{4-3j}{5}$$

$$\frac{3+2j}{(1+2j)} \frac{(1-2j)}{(1-2j)} = \frac{3-6j+2j-4j^2}{1+4} = \frac{3-4j+4}{5} = \frac{7-4j}{5}$$

$$\frac{4-3j-7+4j}{5} \bar{z} = \frac{j-1}{j}$$

$$\frac{-3+j}{5} \bar{z} = \frac{j-1}{j}$$

$$\bar{z} = \frac{(j-1)5}{j(-3+j)} = \frac{5(j-1)(3j-1)}{(-3j-1)(3j-1)} = \frac{5(3j^2-j-3j+1)}{10} = \frac{\cancel{5}^{1}(-2-4j)}{\cancel{10}\ 2}$$

$$\bar{z} = -1-2j \quad \Rightarrow \quad z = -1+2j$$