



Fundamentals - Sum and Product Notation

Exercise 1. Simplify:

$$a) \sum_{k=0}^5 (2k+1) =$$

$$b) \prod_{k=0}^3 (k^2+1) =$$

$$c) \sum_{k=0}^{n-1} (2k+1) - \sum_{k=1}^{n+1} (2k+1) =$$

$$d) \sum_{k=0}^{n-1} (2k+1) - \sum_{k=1}^n (2k-1) =$$

$$a) \sum_{k=0}^5 (2k+1) = \overbrace{1+3+5+7+9+11}^{12} = 12 \cdot \frac{6}{2} = 36$$

$$b) \prod_{k=0}^3 (k^2+1) = 1 \cdot 2 \cdot 5 \cdot 10 = 10 \cdot 10 = 100$$

$$c) \sum_{k=0}^{n-1} (2k+1) - \sum_{k=1}^{n+1} (2k+1) = 1 + \sum_{k=1}^{n-1} (2k+1) - \left(\sum_{k=1}^{n-1} (2k+1) + 2n+1 + 2(n+1)+1 \right)$$

$$(\underbrace{a_0 + a_1 + \dots + a_{n-1}}_{12}) - (\underbrace{a_1 + a_2 + \dots + a_{n-1} + a_n + a_{n+1}}_{12})$$

$$= \cancel{1} - 2n - \cancel{1} - 2n - 2 - 1 = -4n - 3$$

$$d) \sum_{k=0}^{n-1} (2k+1) - \sum_{k=1}^n (2k-1) = \sum_{k=0}^{n-1} (2k+1) - \sum_{k'=0}^{n-1} (2(k'+1)-1) =$$

$$= \sum_{k=0}^{n-1} (2k+1) - \sum_{k=0}^{n-1} (2k+2-1) = \sum_{k=0}^{n-1} (2k+1) - \sum_{k=0}^{n-1} (2k+1) = 0$$