Fundamentals - Sum and Product Notation

Exercise 1. Simplify:

a) $\sum_{k=0}^{5} (2k+1) =$

b) $\prod_{k=0}^{3} (k^2+1) =$

c) $\sum_{k=0}^{n-1} (2k+1) - \sum_{k=1}^{n+1} (2k+1) =$

d) $\sum_{k=0}^{n-1} (2k+1) - \sum_{k=1}^{n} (2k-1) =$

a) $\sum_{k=0}^{5} (2k+1) = \overbrace{1+\underbrace{3+5}_{:2}+7+\underbrace{9+11}_{12}}^{12} = 12 \cdot \frac{6}{2} = 36$

b) $\prod_{k=0}^{3} (k^2+1) = 1 \cdot 2 \cdot 5 \cdot 10 = 10 \cdot 10 = 100$

c) $\sum_{k=0}^{n-1} (2k+1) - \sum_{k=1}^{n+1} (2k+1) = 1 + \sum_{k=1}^{n-1} (2k+1) - \left(\sum_{k=1}^{n-1} (2k+1) + 2n+1+2(n+1)+1 \right)$

$(a_0 + a_1 + \dots + a_{n-1}) \quad -(a_1 + a_2 + \dots + a_{n-1} + a_n + a_{n+1})$

$= 1 - 2n - 1 - 2n - 2 - 1 = -4n - 3$

$k' = k-1 \rightarrow k = k'+1$

d) $\sum_{k=0}^{n-1} (2k+1) - \sum_{k=1}^{n} (2k-1) = \sum_{k=0}^{n-1} (2k+1) - \sum_{k'=0}^{n-1} (2(k'+1)-1) =$

$= \sum_{k=0}^{n-1} (2k+1) - \sum_{k=0}^{n-1} (2k+2-1) = \sum_{k=0}^{n-1} (2k+1) - \sum_{k=0}^{n-1} (2k+1) = 0$