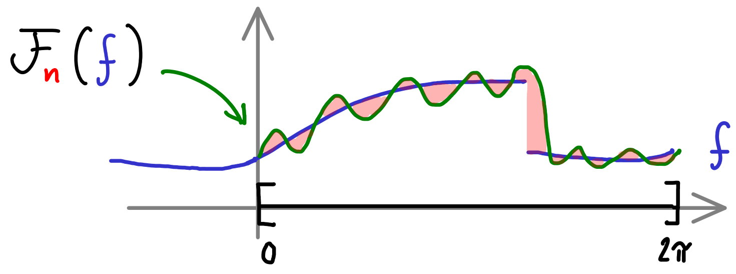
Fourier Transform - Part 14

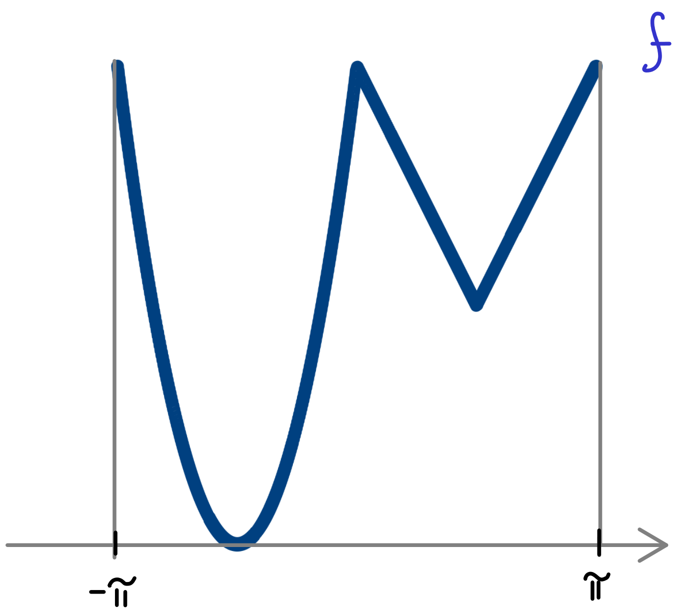


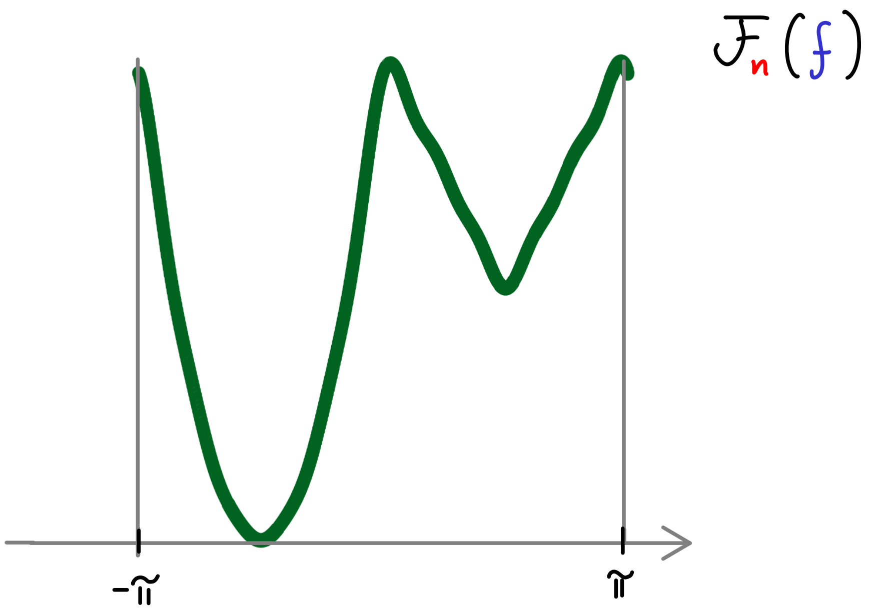
$$\| f - \mathcal{F}_n(f) \|_{L^2} \xrightarrow{n \to \infty} 0$$

not a pointwise convergence!

$\rightsquigarrow$ We can get uniform convergence for special functions

Example: continuous and piecewise C^1-function



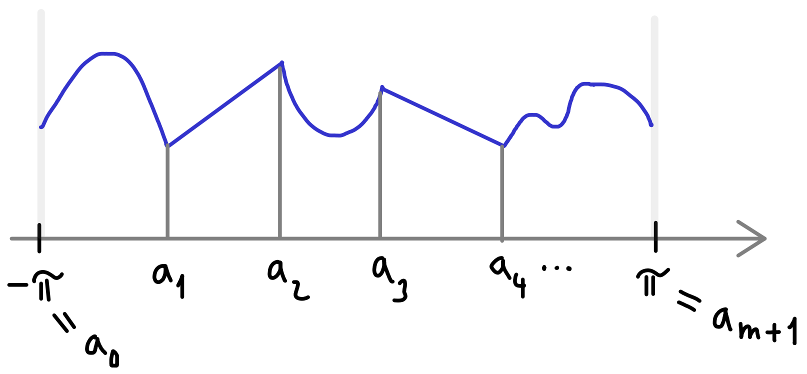


Supremum norm: $\| f \|_\infty := \sup\limits_{x \in [-\pi, \pi]} |f(x)|$

$$\Rightarrow \int_{-\pi}^{\pi} |f(x)|^2 \, dx \leq \int_{-\pi}^{\pi} \| f \|_\infty^2 \, dx = 2\pi \cdot \| f \|_\infty^2$$

$$\Rightarrow \| f \|_{L^2} \leq \| f \|_\infty$$

Theorem: $f: \mathbb{R} \longrightarrow \mathbb{C}$ 2π-periodic continuous function.



Assume there are finitely many points $(a_1, a_2, \dots, a_m)$ inside the interval $[-\pi, \pi]$ such that:

$$f\big|_{[a_j, a_{j+1}]} \in C^1 \quad \text{for all} \quad j \in \{0, 1, \dots, m\}$$

Then: $\| f - \mathcal{F}_n(f) \|_\infty \xrightarrow{n \to \infty} 0$

$$\left(\begin{aligned} \mathcal{F}_n(f) &= \sum_{k=-n}^{n} e_k \langle e_k, f \rangle \\ e_k &: x \mapsto e^{ikx} \\ \langle f, g \rangle &= \frac{1}{2\pi} \int_{-\pi}^{\pi} \overline{f(x)} \cdot g(x)\, dx \end{aligned} \right)$$