Exercise 1. Convergence in a metric space

Let d be the standard metric on the set $X = \mathbb{C}$. Let $a = 1$. Then we define $d_a : X \times X \to \mathbb{R}$:

$$d_a(x, y) := \begin{cases} d(x, a) + d(y, a), & \text{for } x \neq y, \\ 0, & \text{else.} \end{cases}$$

We know that (X, d_a) is also a metric space.

(a) Let $(u_n)_{n \in \mathbb{N}}$ be a sequence that converges to 1 in $(\mathbb{C}, d_a)$. Show that $(u_n)_{n \in \mathbb{N}}$ converges to 1 in $(\mathbb{C}, d)$ as well.

(b) Let $(u_n)_{n \in \mathbb{N}}$ be a sequence that converges to $b \neq 1$ in $(\mathbb{C}, d_a)$ Show that there is an $N \in \mathbb{N}$ such that $u_n = b$ for all $n \geq N$.