## Exercise 1. Convergence in a metric space

Let d be the standard metric on the set  $X = \mathbb{C}$ . Let a = 1. Then we define  $d_a: X \times X \to \mathbb{R}$ :

$$d_a(x,y) := \begin{cases} d(x,a) + d(y,a), & \text{for } x \neq y, \\ 0, & \text{else.} \end{cases}$$

We know that  $(X, d_a)$  is also a metric space.

- (a) Let  $(u_n)_{n \in \mathbb{N}}$  be a sequence that converges to 1 in  $(\mathbb{C}, d_a)$ . Show that  $(u_n)_{n \in \mathbb{N}}$  converges to 1 in  $(\mathbb{C}, d)$  as well.
- (b) Let  $(u_n)_{n \in \mathbb{N}}$  be a sequence that converges to  $b \neq 1$  in  $(\mathbb{C}, d_a)$  Show that there is an  $N \in \mathbb{N}$  such that  $u_n = b$  for all  $n \geq N$ .