

Exercise 1. Absolutely convergent series

Let $(X, \|\cdot\|)$ be a normed space. Show the equivalence of the following statements:

- (a) $(X, \|\cdot\|)$ is a Banach space
- (b) Every absolutely convergent series is convergent, which means for every sequence $(x_n) \subseteq X$ with $\sum_{n=1}^{\infty} \|x_n\| < \infty$, there is an element $x \in X$ such that

$$\left\| x - \sum_{n=1}^N x_n \right\| \xrightarrow{N \rightarrow \infty} 0$$