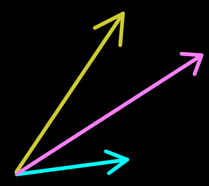
Functional analysis - part 6



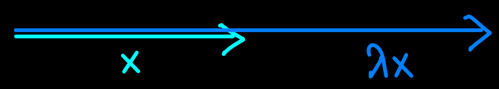
Definition: $\mathbb{F} \in \{\mathbb{R}, \mathbb{C}\}$. Let X be a $\mathbb{F}$-vector space.

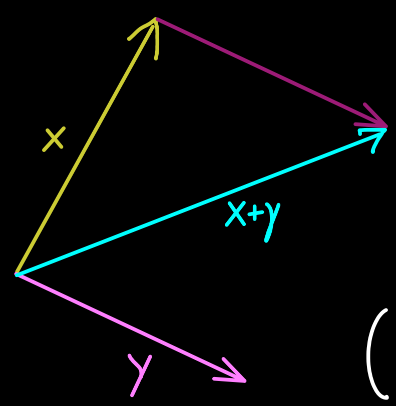
A map $\|\cdot\| : X \longrightarrow [0, \infty)$ is called <u>norm</u> if

(a) $\|x\| = 0 \iff x = 0$ (positive definite)

(b) $\|\lambda \cdot x\| = |\lambda| \, \|x\|$ for all $\lambda \in \mathbb{F}$, $x \in X$ (absolutely homogeneous)

absolute value in $\mathbb{R}$ or $\mathbb{C}$





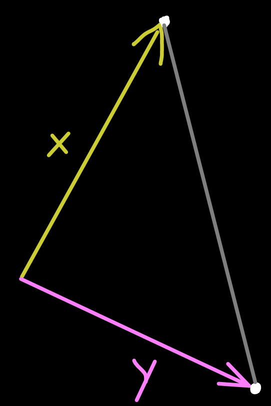
(c) $\|x + y\| \leq \|x\| + \|y\|$ for all $x, y \in X$ (triangle inequality)

$(X, \|\cdot\|)$ is then called a <u>normed space</u>.

Important: If $\|\cdot\|$ is a norm for the $\mathbb{F}$-vector space X, then

$$d_{\|\cdot\|}(x,y) := \|x - y\|$$ defines

a metric for the set X.



If $(X, d_{\|\cdot\|})$ is a <u>complete</u> metric space,

then the normed space $(X, \|\cdot\|)$ is called a <u>Banach space</u>.

Banach space:

complete metric space	real / complex vector space

norm $\|\cdot\|$