The Bright Side of Mathematics

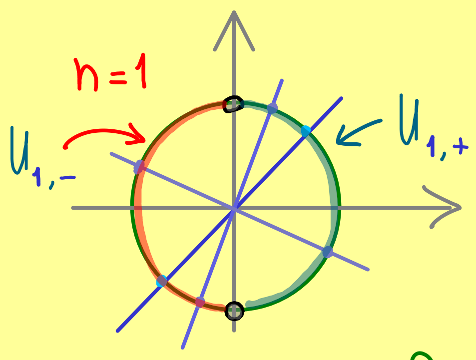
Manifolds – Part 11

$$S^n := \{x \in \mathbb{R}^{n+1} \mid \|x\| = 1\}$$

$$\{x \in \mathbb{R}^{n+1} \mid \pm x_i > 0\}$$

is an n-dimensional manifold with atlas $\left(U_{i,\pm},\, h_{i,\pm}\right)_{i\in\{1,\dots,n+1\}}$

Projective space: $P^n(\mathbb{R}) := S^n/\sim$ with quotient topology



equivalence relation: $x \sim y :\Longleftrightarrow (x = y \text{ or } x = -y)$

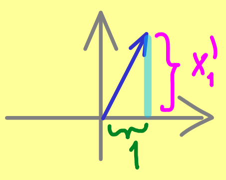
$$q: S^n \to S^n/\sim, \quad x \mapsto [x]_\sim \qquad \text{canonical projection}$$

$$V_i := \{[x]_\sim \in P^n(\mathbb{R}) \mid x_i \neq 0\}, \quad q^{-1}[V_i] = U_{i,+} \cup U_{i,-}$$

open

for $n = 1$: $h_1 : V_1 \to V_1' \subseteq \mathbb{R}^1$, $h_1([x]_\sim) = \frac{x_2}{x_1}$ slope

with inverse $h_1^{-1}(x_1') = \left[\begin{pmatrix} 1 \\ x_1' \end{pmatrix} \cdot \frac{1}{\sqrt{1^2 + (x_1')^2}}\right]_\sim$



h_2 works similarly $\Longrightarrow$ 1-dimensional manifold

for $n \in \mathbb{N}$: $h_i : V_i \to V_i' \subseteq \mathbb{R}^n$

$$h_i([x]_\sim) = \begin{pmatrix} \frac{x_1}{x_i} \\ \vdots \\ \frac{x_{i-1}}{x_i} \\ \frac{x_{i+1}}{x_i} \\ \vdots \\ \frac{x_{n+1}}{x_i} \end{pmatrix} \qquad \text{homeomorphism}$$

$\Longrightarrow$ n-dimensional manifold