BECOME A MEMBER

ON STEADY

The Bright Side of
Mathematics

Manifolds – Part 12

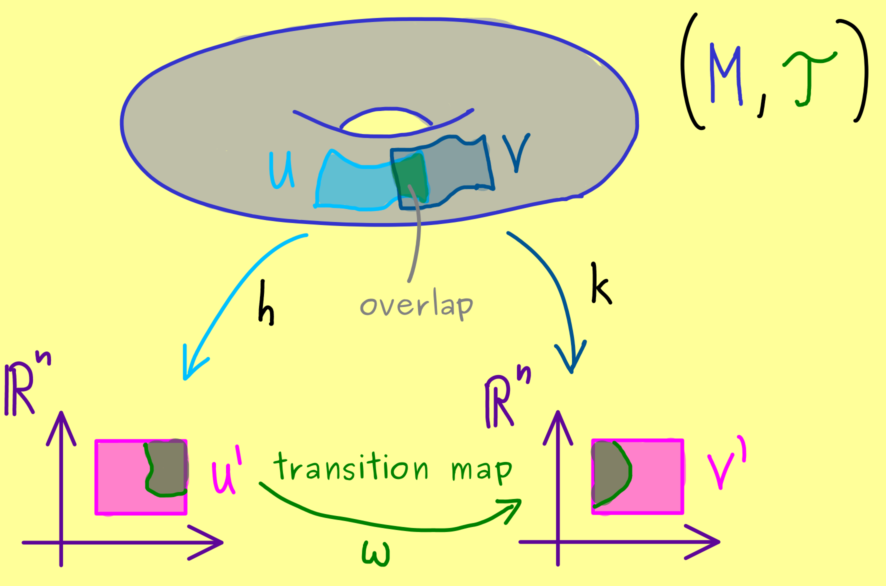
Smooth structures

C^k-diffeomorphism

$\mathbb{R}^n \longrightarrow \mathbb{R}^n$

ω : $\longrightarrow$

$h[U \cap V]$ $\quad$ $k[U \cap V]$



C^k-diffeomorphism : $k \in \{0, 1, \dots\}$ or $k = \infty$

- ω is k-times continuously differentiable (partial derivatives up to the k-th order exist and are continuous) $\Big\}\ \omega \in C^k(\cdot)$
- ω is bijective
- $\omega^{-1} \in C^k(\cdot\cdot)$

Definition:

- Two charts h, k are called C^k- smoothly compatible if the transition map is a C^k-diffeomorphism.
- An atlas $\{(U_i, h_i)_{i \in I}\}$ is called a C^k-atlas if any two charts are C^k- smoothly compatible.
- A maximal C^k-atlas $\mathcal{A}$ is: (1) $\mathcal{A}$ is a C^k-atlas
  (2) For any other C^k-atlas $\mathcal{B}$, we have $\mathcal{B} \not\supset \mathcal{A}$.

Definition: n-dimensional C^k-smooth manifold:

- n-dimensional (topological) manifold
- maximal C^k-atlas $\big(C^k$-smooth structure$\big)$