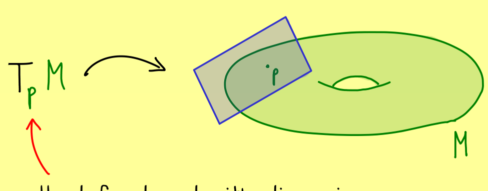
The Bright Side of Mathematics

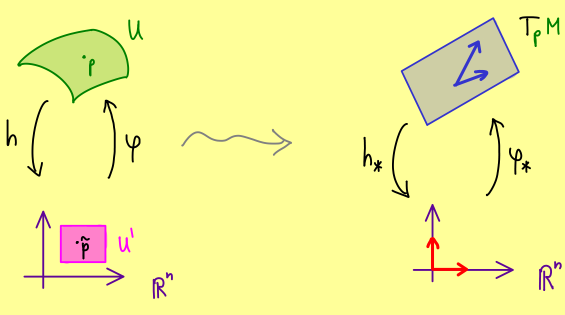
Manifolds – Part 22

smooth manifold M of dimension n, $p \in M$.



well-defined and with dimension n

chart (U, h):



defined by:

$h_* : T_p M \to \mathbb{R}^n$

$[\gamma] \mapsto (h \circ \gamma)'(0)$

linear + bijective

$\varphi_* := h_*^{-1}$

<u>Definition:</u> <u>coordinate basis</u> (standard basis with respect to (U, h)):

For (U, h) and $p \in U$, we define: $\partial_j := \varphi_*(e_j)$

where $(e_1, e_2, \ldots, e_n)$ is the standard basis of $\mathbb{R}^n$

<u>Remember:</u> For submanifolds:

$$\begin{array}{ccc} T_p M & \longrightarrow & T_p^{sub} M \\ \uparrow \varphi_* & \nearrow J_\varphi(\tilde{p}) & \\ \mathbb{R}^n & & \end{array}$$

$(\partial_1, \partial_2, \ldots, \partial_n)$ is essentially $\left(\frac{\partial \varphi}{\partial x_1}(\tilde{p}), \frac{\partial \varphi}{\partial x_2}(\tilde{p}), \ldots, \frac{\partial \varphi}{\partial x_n}(\tilde{p}) \right)$

<u>Soon:</u> $f: M \to N$ smooth $\rightsquigarrow$ $df_p : T_p M \to T_p N$ differential