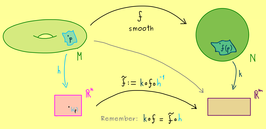
Manifolds – Part 24

Differential in local charts?



Choose: $[\gamma] \in T_p M$:

$$d_{k(f(p))}\big(df_p([\gamma])\big) = d_{k(f(p))}\big([f\circ\gamma]\big)$$
$$= \big[k \circ f \circ \gamma\big] = \big(k \circ f \circ \gamma\big)'(0)$$
$$= \big(\tilde{f} \circ h \circ \gamma\big)'(0)$$

ordinary chain rule

$$= J_{\tilde{f}}(\ldots)\,\big(h\circ\gamma\big)'(0)$$
$$\overset{\text{tangent}}{=} J_{\tilde{f}}(\ldots)\,[h\circ\gamma]$$
$$= J_{\tilde{f}}(\ldots)\; d_{h\circ\gamma}[\gamma]$$

Remember:

$$f = k^{-1} \circ \tilde{f} \circ h$$
$$df = dk^{-1} J_{\tilde{f}}\, dh$$