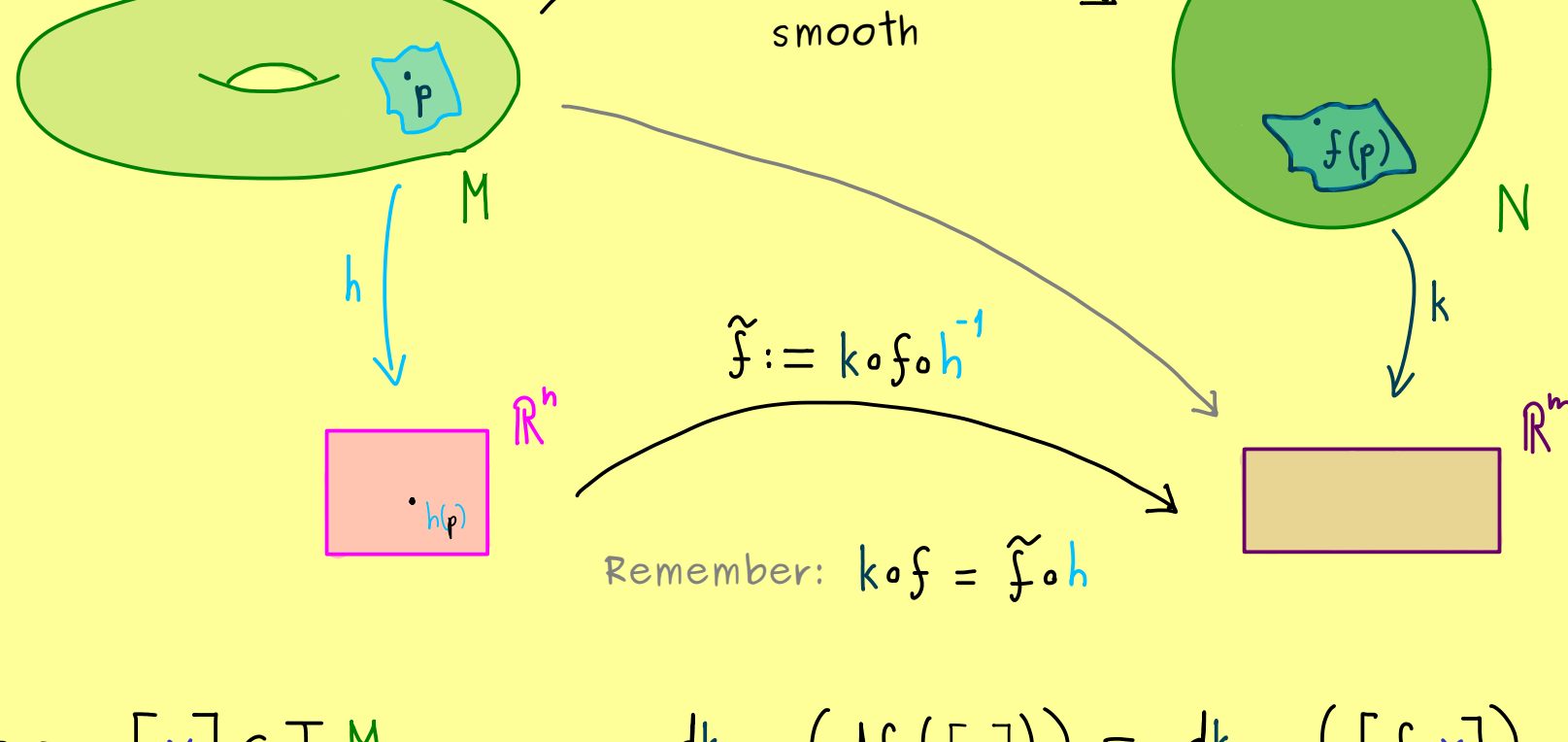




Manifolds - Part 24

Differential in local charts?



Choose: $[\gamma] \in T_p M$:

$$\begin{aligned}
 dk_{f(p)}(df_p([\gamma])) &= dk_{f(p)}([f \circ \gamma]) \\
 &= [k \circ f \circ \gamma] \stackrel{\text{bijection}}{=} (k \circ f \circ \gamma)'(0) \\
 &= (\tilde{f} \circ h \circ \gamma)'(0) \\
 &\stackrel{\text{ordinary chain rule}}{=} J_{\tilde{f}}(h(p)) (h \circ \gamma)'(0) \\
 &\stackrel{\text{bijection}}{=} J_{\tilde{f}}(h(p)) [h \circ \gamma] \\
 &= J_{\tilde{f}}(h(p)) dh_p([\gamma])
 \end{aligned}$$

Remember:

$$\begin{aligned}
 f &= k^{-1} \circ \tilde{f} \circ h \\
 df &= dk^{-1} J_{\tilde{f}} dh
 \end{aligned}$$