Ordinary Differential Equations - Part 3

ODE: $\dot{x} = w(t, x)$ (explicit, of first order)

Example: (a) $\dot{x} = \lambda \cdot x$ $\rightsquigarrow$ autonomous

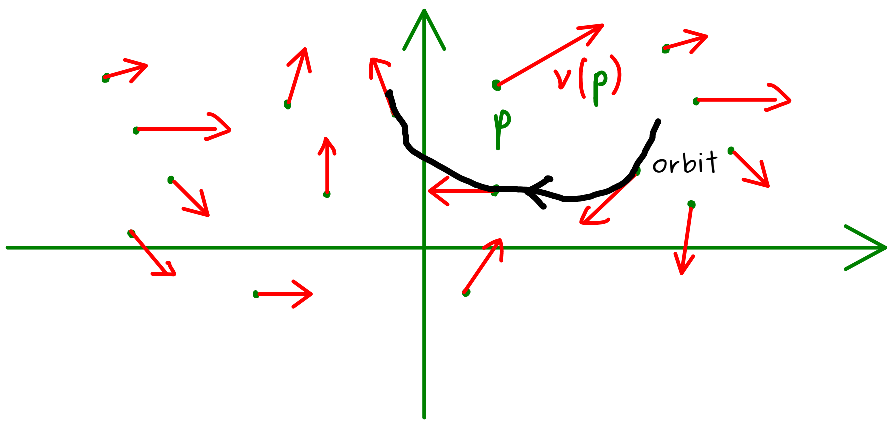
(b) $\dot{x} = t$ $\rightsquigarrow$ not autonomous

(c) $\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} x_2 \\ -x_1 \end{pmatrix}$ $\rightsquigarrow$ autonomous

Definition: autonomous system: $\dot{x} = v(x)$ with $v : U \rightarrow \mathbb{R}^n$, $U \subseteq \mathbb{R}^n$

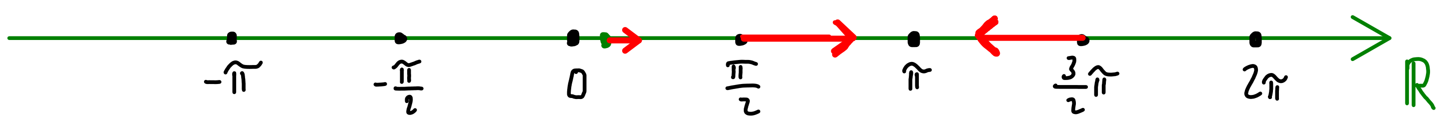
often:
U open
v continuous

Directional field:



$v : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$

Examples: (a) $\dot{x} = \sin(x)$, $v : \mathbb{R} \rightarrow \mathbb{R}$, $v(x) = \sin(x)$



(1) $\alpha(t) = 0$ for all $t \in \mathbb{R}$ is a solution: $\underbrace{\dot{\alpha}(t)}_{=0} = \underbrace{\sin(\alpha(t))}_{=0}$

(2) $\alpha(t) = \pi$ for all $t \in \mathbb{R}$ is a solution.

(3) A solution with $\alpha(0) = \frac{\pi}{2}$ is monotonically increasing with $\lim\limits_{t \to \infty} \alpha(t) = \pi$.

(b) $\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} -x_2 \\ x_1 \end{pmatrix}$, $V: \mathbb{R}^2 \to \mathbb{R}^2$, $(x_1, x_2) \mapsto \begin{pmatrix} -x_2 \\ x_1 \end{pmatrix}$