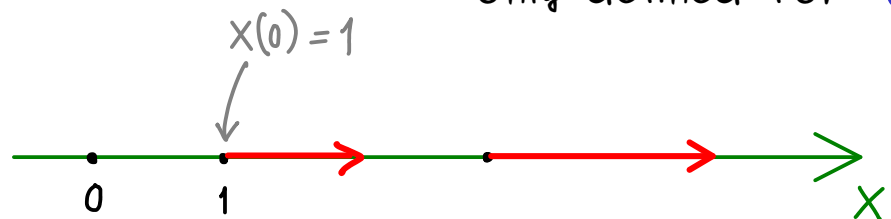# Ordinary Differential Equations — Part 8

<u>Questions:</u> Initial value problem: $\dot{x} = V(x)$ with $V: \mathbb{R}^n \longrightarrow \mathbb{R}^n$ continuous
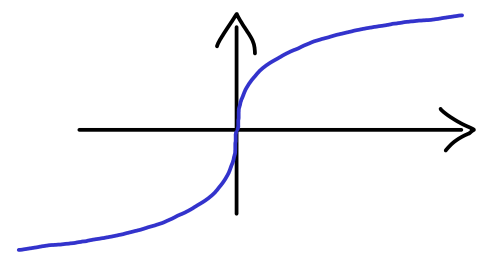
$$x(0) = x_0$$

- Does a solution exist?

- What is the domain of definition?

- Uniqueness of solutions?

<u>Examples:</u> (a) $\dot{x} = x^2$, $x(0) = 1$ $\overset{\text{part 5}}{\Longrightarrow}$ solution exists: $\alpha(t) = \dfrac{1}{1-t}$

only defined for $t < 1$

$X(0) = 1$

(b) $\dot{x} = V(x)$, $x(0) = 0$ with $V(x) = \begin{cases} \sqrt{|x|} & , \ x \geq 0 \\ -\sqrt{|x|} & , \ x < 0 \end{cases}$
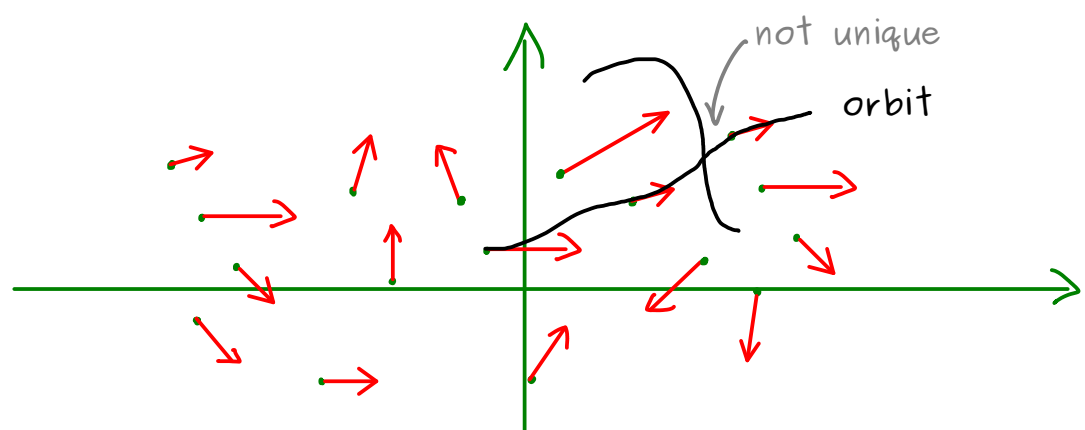

We find at least two solutions: $\alpha(t) = 0$ for all t

$$\tilde{\alpha}(t) = \begin{cases} 0 & , \ t \leq 0 \\ \dfrac{1}{4}t^2 & , \ t > 0 \end{cases}$$

<u>In general:</u>

directional field

not unique
orbit

existence: does each point have an orbit?

uniqueness: can two orbits cross?