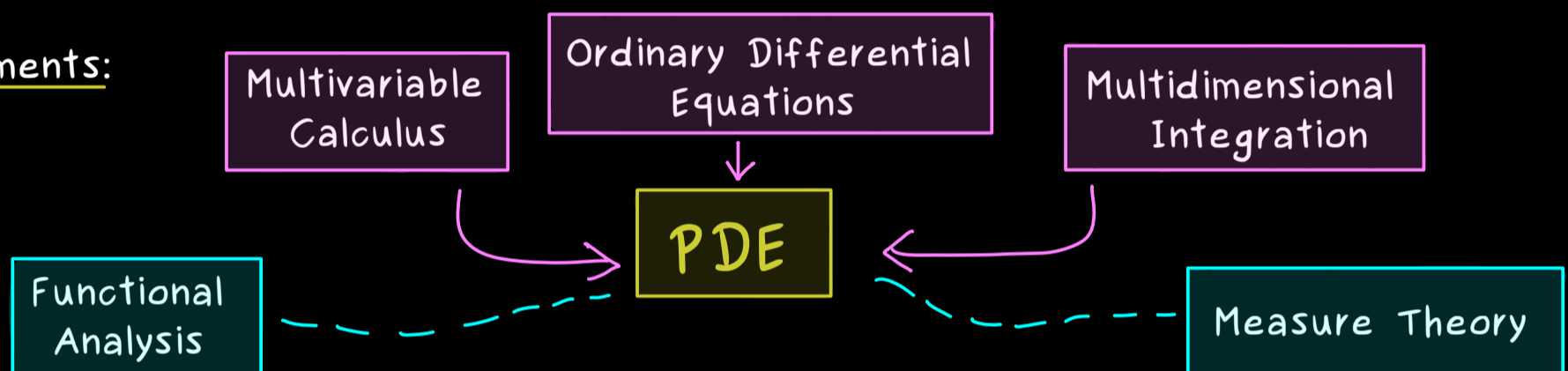


Partial Differential Equations

- Goal: Understanding: (1) Laplace's equation $\Delta f = 0$ (elliptic)
(2) Heat equation $\Delta f = \frac{\partial}{\partial t} f$ (parabolic)
(3) Wave equation $\Delta f = \frac{\partial^2}{\partial t^2} f$ (hyperbolic)

After that: abstract theory (Sobolev spaces, distributions, ...)

Requirements:



What is a PDE?

A partial differential equation (PDE) is an equation for an unknown function $u: \Omega \rightarrow \mathbb{R}$ (where $\Omega \subseteq \mathbb{R}^n$ open, $n \geq 2$)

involving some partial derivatives of u :

$$F\left(x, u(x), \left(\mathcal{D}^{\alpha} u\right)(x)\right) = 0 \quad (*)$$

multi-index
 $1 \leq |\alpha| \leq m$

Example:

$$\sin(u(x)) + \frac{\partial u}{\partial x_1}(x) \cdot \frac{\partial^2 u}{\partial x_1 \partial x_2}(x) = 0 \quad \text{order} = 2$$

$$\sum_{|\alpha| \leq m} a_{\alpha}(x) \cdot \mathcal{D}^{\alpha} u(x) = f(x) \quad \text{(linear PDE)}$$

Definition:

A (classical) solution of the PDE (*) is a function $u: \Omega \rightarrow \mathbb{R}$ where all partial derivatives are well-defined and

$$F\left(x, u(x), \left(\mathcal{D}^{\alpha} u\right)(x)\right) = 0 \quad \text{for all } x \in \Omega.$$

$1 \leq |\alpha| \leq m$