Partial Differential Equations

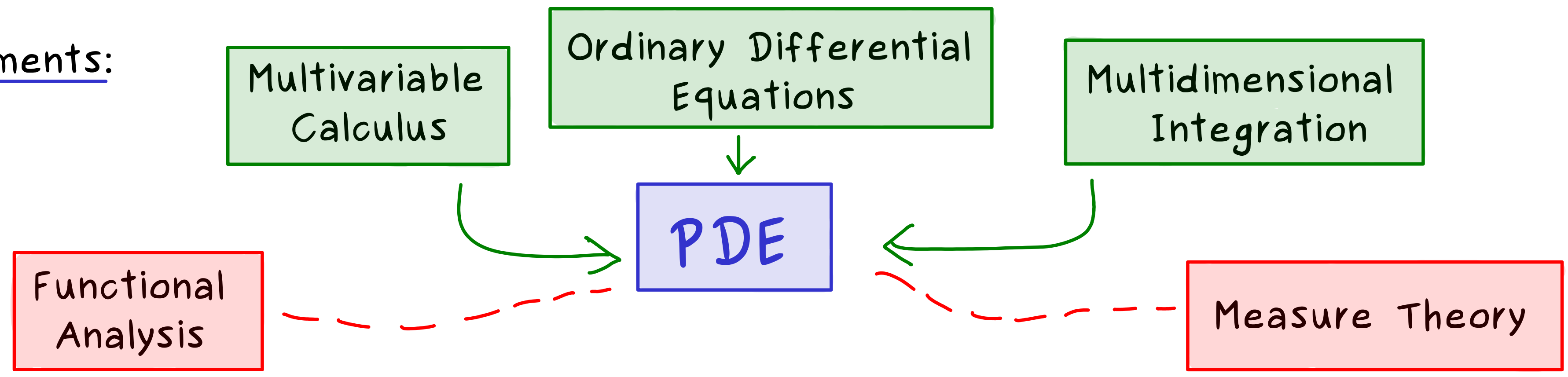
Goal: Understanding: (1) Laplace's equation $\Delta f = 0$ (elliptic)

(2) Heat equation $\Delta f = \frac{\partial}{\partial t} f$ (parabolic)

(3) Wave equation $\Delta f = \frac{\partial^2}{\partial t^2} f$ (hyperbolic)

After that: abstract theory (Sobolev spaces, distributions, ...)

Requirements:



What is a PDE? A partial differential equation (PDE) is an equation for an unknown function $u : \Omega \longrightarrow \mathbb{R}$ (where $\Omega \subseteq \mathbb{R}^n$ open, $n \geq 2$) involving some partial derivatives of u:

multi-index

$$F\Big(x, u(x), (D^\alpha u)_{1 \leq |\alpha| \leq m}(x)\Big) = 0 \qquad (*)$$

Example:

$$\sin(u(x)) + \frac{\partial u}{\partial x_1}(x) \cdot \frac{\partial^2 u}{\partial x_1 \partial x_2}(x) = 0 \qquad \text{order} = 2$$

$$\sum_{|\alpha| \leq m} a_\alpha(x) \cdot D^\alpha u(x) = f(x) \qquad \text{(linear PDE)}$$

Definition: A (classical) solution of the PDE $(*)$ is a function $u : \Omega \longrightarrow \mathbb{R}$ where all partial derivatives are well-defined and

$$F\Big(x, u(x), (D^\alpha u)_{1 \leq |\alpha| \leq m}(x)\Big) = 0 \quad \text{for all } x \in \Omega.$$