The Bright Side of
Mathematics

Probability Theory – Part 17

standard deviation $= \sqrt{\text{variance}}$

Definition: $(\Omega, \mathcal{A}, \mathbb{P})$ probability space, $X: \Omega \to \mathbb{R}$ random variable,

where $\int_\Omega X^2 \, d\mathbb{P}$ exists. Then:

$$\sigma(X) = \sqrt{\mathrm{Var}(X)}$$

is called the standard deviation of X.

$$\sigma(X) = \sqrt{\mathbb{E}(X^2) - \mathbb{E}(X)^2}$$

Examples: (a) $X \sim \mathrm{Uniform}(\{x_1, x_2, \dots, x_n\})$ discrete case with $\mathbb{P}_X(\{x_i\}) = \frac{1}{n}$

$$\sigma(X) = \sqrt{\frac{1}{n} \cdot \sum_{j=1}^{n} (x_j - \overline{x})^2}$$

(b) $X \sim \mathrm{Normal}(\mu, \sigma^2)$ continuous case with pdf

$$f_X(x) = \frac{1}{\sigma \cdot \sqrt{2\pi}} \cdot e^{-\frac{1}{2} \cdot \left(\frac{x-\mu}{\sigma}\right)^2}$$

$$\mathbb{E}(X) = \mu$$

$$\sigma(X) = \sigma$$