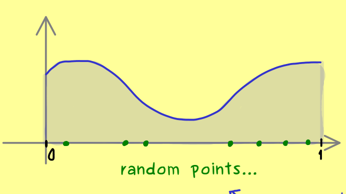
The Bright Side of
Mathematics

Probability Theory - Part 29

<u>law of large numbers:</u> n repetitions $X_1, X_2, \dots, X_n$: $\frac{1}{n}\sum_{k=1}^{n} X_k \xrightarrow{n\to\infty} \mathbb{E}(X)$

"Monte Carlo method"

<u>Monte Carlo integration:</u>



random variables: $X_1, X_2, \dots$

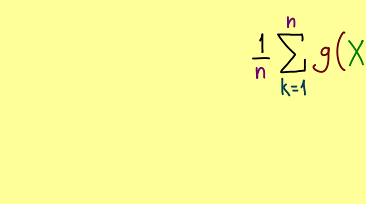
<u>Weak law of large numbers:</u> $Y_k : \Omega \longrightarrow \mathbb{R}$ random variables.

Let $(Y_k)_{k\in\mathbb{N}}$ be i.i.d. and $\mathbb{E}(|Y_1|) < \infty$.

Then for $\mu := \mathbb{E}(Y_1)$ and for all $\varepsilon > 0$: $\mathbb{P}\left(\left| \frac{1}{n}\sum_{k=1}^{n} Y_k - \mu \right| \geq \varepsilon \right) \xrightarrow{n\to\infty} 0$

<u>Monte Carlo integration:</u> Given: $g : [0,1] \longrightarrow [-c,c]$ integrable, $c > 0$.

We want: $\int_0^1 g(x)\, dx$



Take: X_1 picks a point (randomly = uniformly distributed)

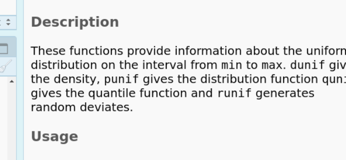
from the interval $[0,1]$: $x_1 = X_1(\omega)$

$Y_1 := g(X_1)$ What is $\mathbb{E}(Y_1)$?

$$\mathbb{E}(Y_1) = \mathbb{E}(g(X_1)) \overset{\text{change of variables}}{=} \int_0^1 g(x) \underbrace{f_{X_1}(x)}_{=1,\ \text{pdf}} dx = \int_0^1 g(x)\, dx$$

procedure: $X_1, X_2, \dots$ i.i.d.+uniformely distributed on $[0,1]$

$\frac{1}{n}\sum_{k=1}^{n} g(X_k)$ approximates $\int_0^1 g(x)\,dx$



<u>Example:</u>

$$\int_0^1 \frac{4}{1+x^2}\, dx = \pi$$