Exercise 1. Antiderivative

(a) Consider the function $f\colon [0,\infty) \to \mathbb{R}$ *defined by*

$$f(x) := \begin{cases} x^{3/2}\sin\left(\frac{1}{x}\right) & \text{if } x > 0, \\ 0 & \text{if } x = 0. \end{cases}$$

(i) Show that f *is differentiable on* $[0,\infty)$ *and compute the derivative* f' *of the function* f.

(ii) Let $c > 0$. *Show that* f' *is Riemann integrable on every interval of the form* $[\varepsilon, c]$ *with* $0 < \varepsilon < c$. *Why is* f' *not Riemann integrable on intervals of the form* $[0, c]$*?*

(b) Consider the function $g\colon [-1,1] \to \mathbb{R}$ *defined by*

$$g(x) := \begin{cases} -1 & \text{if } x \in [-1,0), \\ 1 & \text{if } x \in [0,1]. \end{cases}$$

(i) Show that g *is Riemann integrable on* $[-1,1]$.

(ii) Show that g *does not have an antiderivative on* $[-1,1]$.

Exercise 2. Simple Integrals

Compute the following indefinite integrals (meaning: antiderivatives) using the Fundamental Theorem of Calculus:

(a) $\int \frac{x^2}{1+x^2}\,dx$

(b) $\int \tan(x)\,dx$

(c) $\int \cot(x)\,dx$

(d) $\int \frac{1}{x\log(x)}\,dx$

(e) $\int xe^{-x^2}\,dx$

(f) $\int e^{e^x+x}\,dx$