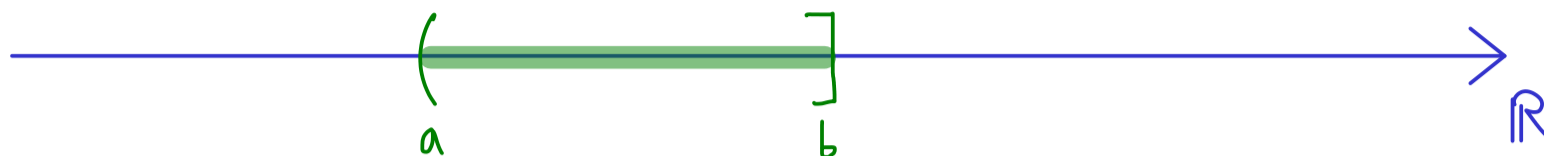


Real Analysis - Part 6

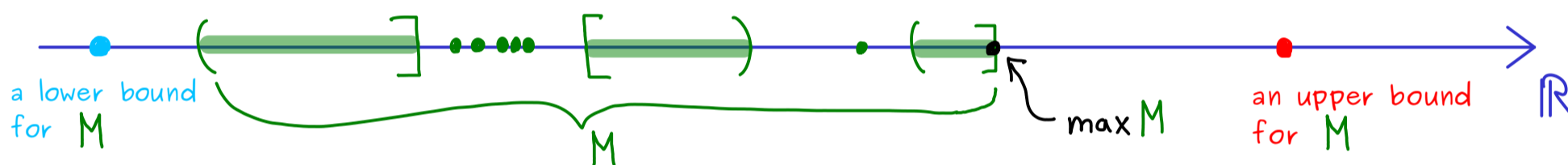


interval: $(a, b] := \{x \in \mathbb{R} \mid a < x \leq b\}$

$$[a, b] := \{x \in \mathbb{R} \mid a \leq x \leq b\}$$

$$[a, \infty) := \{x \in \mathbb{R} \mid a \leq x\}$$

$$(-\infty, b) := \{x \in \mathbb{R} \mid x < b\}$$



Definition: For a subset $M \subseteq \mathbb{R}$: $b \in \mathbb{R}$ is called an upper bound for M if

$$\forall x \in M : x \leq b$$

$a \in \mathbb{R}$ is called a lower bound for M if $\forall x \in M : x \geq a$

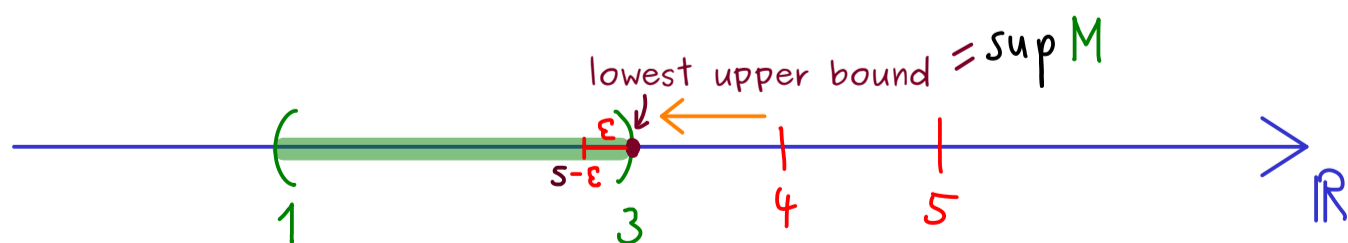
If b is an upper bound for M and $b \in M$, then b is called a maximal element of M.

If a is a lower bound for M and $a \in M$, then a is called a minimal element of M.

$\min M$

Example: • $M = [1, 3]$, $\max M = 3$ $\min M = 1$

• $M = (1, 3)$, $\max M, \min M$ do not exist $\rightsquigarrow \sup M, \inf M$



Definition: For a subset $M \subseteq \mathbb{R}$: $s \in \mathbb{R}$ is called supremum of M if

- $\forall x \in M : x \leq s$ (upper bound for M)
- $\forall \varepsilon > 0 \exists \tilde{x} \in M : s - \varepsilon < \tilde{x}$ ($s - \varepsilon$ is no upper bound for M)

Then write: $\sup M := s$ or $\sup M := \infty$ if M is not bounded from above
or $\sup \emptyset := -\infty$

For a subset $M \subseteq \mathbb{R}$: $l \in \mathbb{R}$ is called infimum of M if

- $\forall x \in M : x \geq l$ (lower bound for M)
- $\forall \varepsilon > 0 \exists \tilde{x} \in M : l + \varepsilon > \tilde{x}$ ($l + \varepsilon$ is no lower bound for M)

Then write: $\inf M := l$ or $\inf M := -\infty$ if M is not bounded from below
or $\inf \emptyset := \infty$