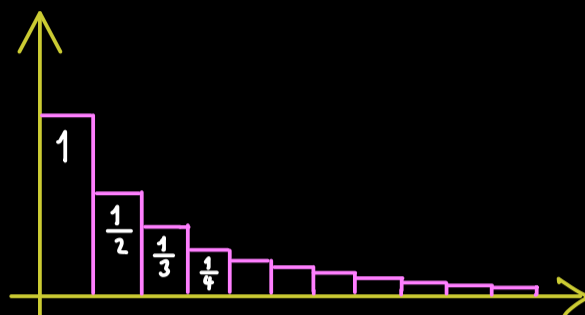


## Real Analysis - Part 18

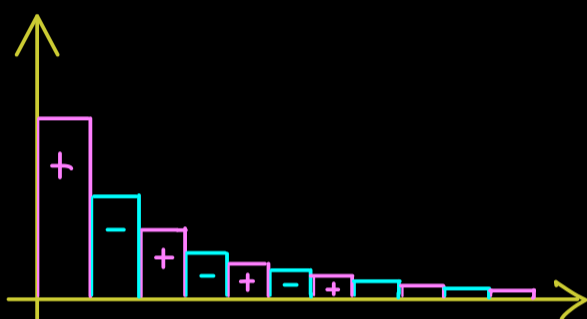
Harmonic series:

$$S_n = \sum_{k=1}^n \frac{1}{k}$$

divergent



Leibniz criterion:



$$S_n = \sum_{k=1}^n (-1)^{k+1} \frac{1}{k}$$

convergent

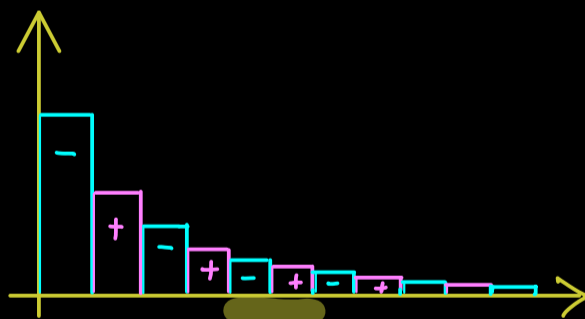
Theorem: (Alternating series test, Leibniz criterion, Leibniz's test)

Let  $(a_k)_{k \in \mathbb{N}}$  be convergent with  $\lim_{k \rightarrow \infty} a_k = 0$  and monotonically decreasing.

Then:  $\sum_{k=1}^{\infty} (-1)^k a_k$  is convergent.

Proof:  $S_n = \sum_{k=1}^n (-1)^k a_k$

$$\Rightarrow a_k \geq 0$$

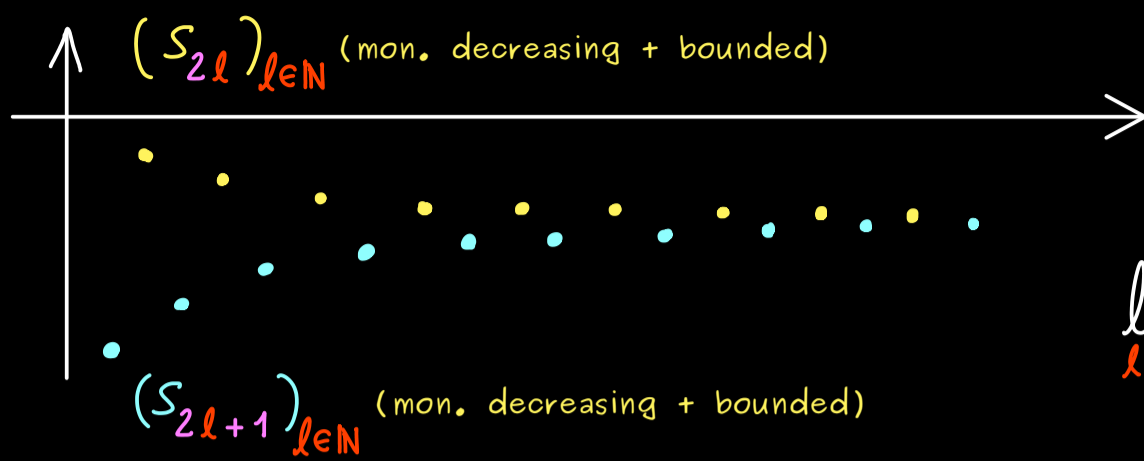


$$S_{2l+2} - S_{2l} = -a_{2l+1} + a_{2l+2} \leq 0 \quad (\text{monotonically decreasing})$$

$$S_{2l+3} - S_{2l+1} = a_{2l+2} - a_{2l+3} \geq 0 \quad (\text{monotonically increasing})$$

$$S_{2l+1} - S_{2l} = -a_{2l+1} \leq 0 \quad \Rightarrow \quad S_3 \leq S_{2l+1} \leq S_{2l} \leq S_2$$

(bounded)



$$\lim_{l \rightarrow \infty} (s_{2l+1} - s_{2l}) =$$

$$\lim_{l \rightarrow \infty} (-a_{2l+1}) = 0$$

$$s := \lim_{l \rightarrow \infty} s_{2l+1} = \lim_{l \rightarrow \infty} s_{2l} \Rightarrow \lim_{n \rightarrow \infty} s_n = s \quad (\text{convergent!})$$

Example:

$$\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt{k}} \quad \text{convergent by Leibniz criterion}$$