

Real Analysis - Part 24

sequence of functions:

$$(f_1, f_2, f_3, f_4, f_5, \dots)$$

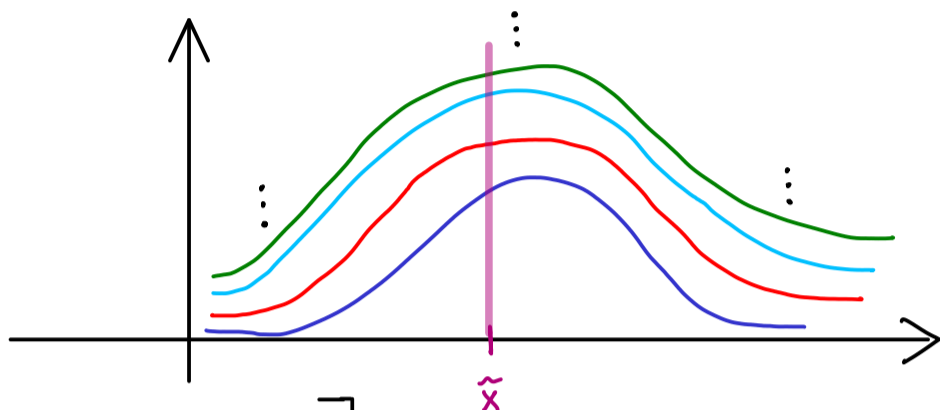
$$f_n: I \rightarrow \mathbb{R}$$

Pointwise convergence: $(f_1, f_2, f_3, f_4, f_5, \dots)$ is pointwisely convergent to a function

$$f: I \rightarrow \mathbb{R} \text{ if for all } \tilde{x} \in I:$$

$$(f_1(\tilde{x}), f_2(\tilde{x}), f_3(\tilde{x}), f_4(\tilde{x}), f_5(\tilde{x}), \dots)$$

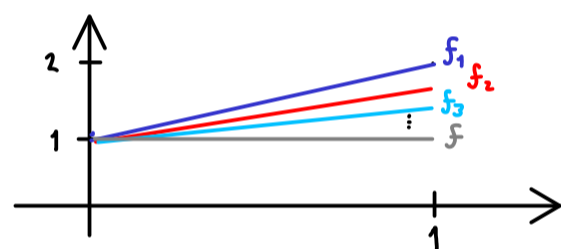
is convergent to $f(\tilde{x})$.



$$\left[\forall \tilde{x} \in I \quad \forall \epsilon > 0 \quad \exists N \in \mathbb{N} \quad \forall n \geq N: |f_n(\tilde{x}) - f(\tilde{x})| < \epsilon \right]$$

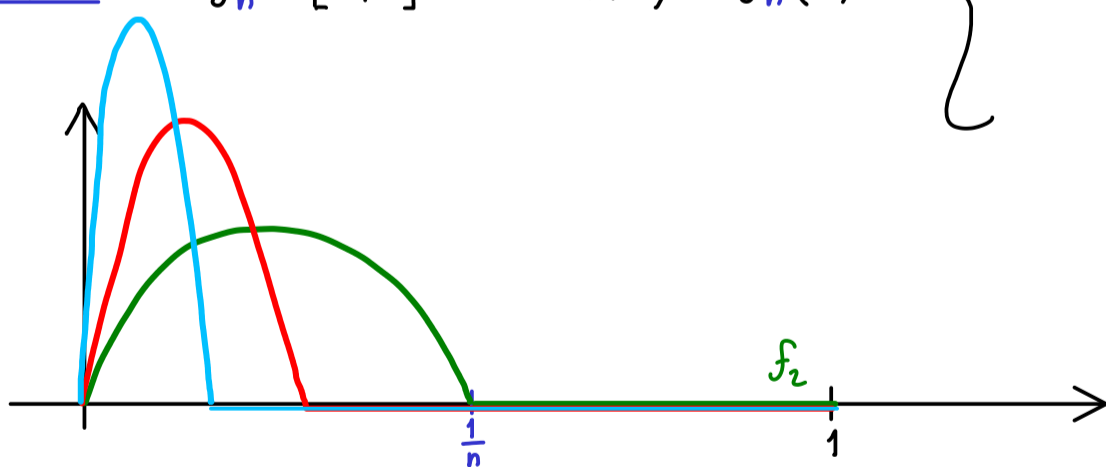
Example: $f_n: [0, 1] \rightarrow \mathbb{R}, f_n(x) = \frac{1}{n}x + 1$

$$\text{For } \tilde{x} \in [0, 1]: f_n(\tilde{x}) = \frac{1}{n}\tilde{x} + 1 \xrightarrow{n \rightarrow \infty} 1$$



\Rightarrow (pointwise) limit function $f: [0, 1] \rightarrow \mathbb{R}, f(x) = 1$

Example: $f_n: [0, 1] \rightarrow \mathbb{R}, f_n(x) = \begin{cases} n^2 x(1-nx) & , x \in [0, \frac{1}{n}] \\ 0 & , x \in (\frac{1}{n}, 1] \end{cases}$



$$f_n\left(\frac{1}{2} \cdot \frac{1}{n}\right) = n^2 \cdot \frac{1}{2n} \cdot \left(1 - n \cdot \frac{1}{2n}\right) = \frac{n}{4}$$

For $x = 0$: $f_n(x) = 0$ for all $n \in \mathbb{N}$
 For $x > 0$: $f_n(x) = 0$ for all $n > \frac{1}{x}$

\Rightarrow (pointwise) limit function $f: [0, 1] \rightarrow \mathbb{R}, f(x) = 0$

Example:

