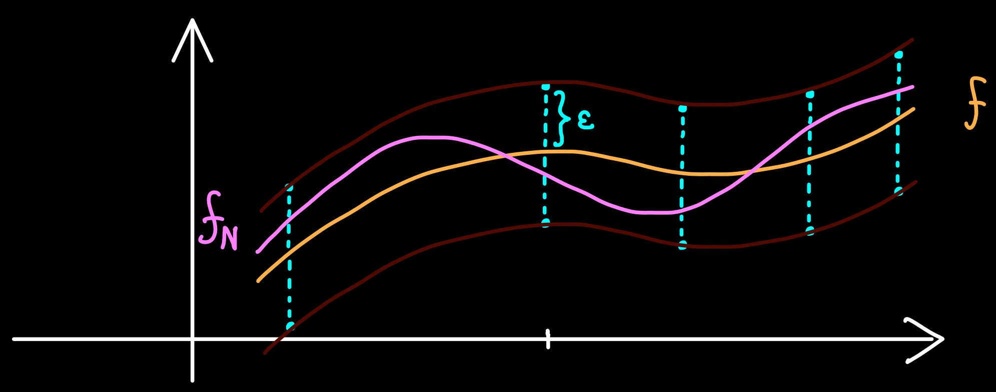
Real Analysis – Part 25

$(f_1, f_2, f_3, f_4, f_5, \dots)$ is pointwisely convergent to $f: I \to \mathbb{R}$

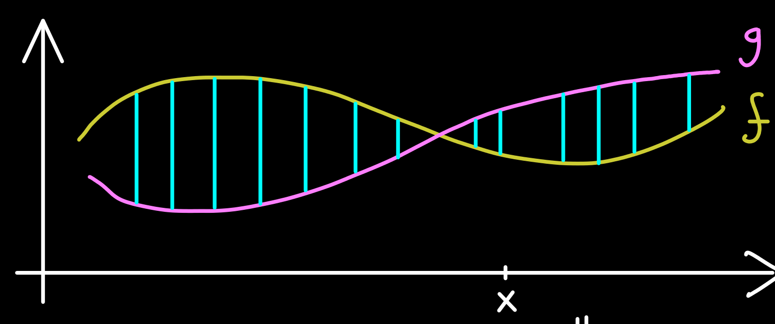
$$\forall \tilde{x} \in I \quad \forall \varepsilon > 0 \quad \exists N_{\tilde{x}} \in \mathbb{N} \quad \forall n \geq N : \quad |f_n(\tilde{x}) - f(\tilde{x})| < \varepsilon$$

Definition: $(f_1, f_2, f_3, f_4, f_5, \dots)$ is uniformly convergent to $f: I \to \mathbb{R}$ if

$$\forall \varepsilon > 0 \quad \exists N \in \mathbb{N} \quad \forall n \geq N \quad \forall \tilde{x} \in I \quad : \quad |f_n(\tilde{x}) - f(\tilde{x})| < \varepsilon$$



Distance for functions:



$f: I \to \mathbb{R}$

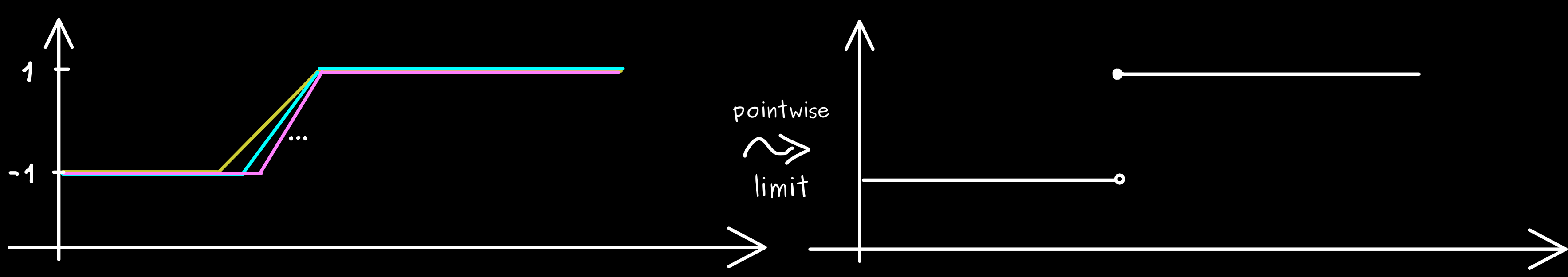
$g: I \to \mathbb{R}$

supremum norm of $f - g$

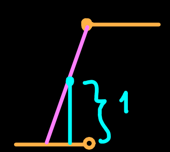
$$\|f - g\|_\infty = \sup_{x \in I} |f(x) - g(x)|$$

$$\boxed{\text{Uniform convergence means: } \quad \|f_n - f\|_\infty \xrightarrow{n \to \infty} 0}$$

Example:



$$\|f_n - f\|_\infty \geq 1 \quad \text{for all } n$$



Result pointwise convergence $\not\Rightarrow$ uniform convergence

$\Leftarrow$