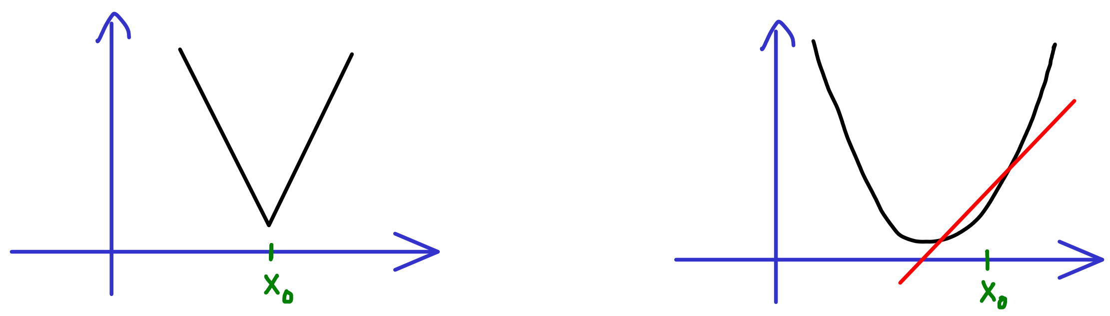
Real Analysis – Part 34

Differentiability (linearisation) smoothness

$f : \mathbb{R} \to \mathbb{R}$



slope at point x_0 ?

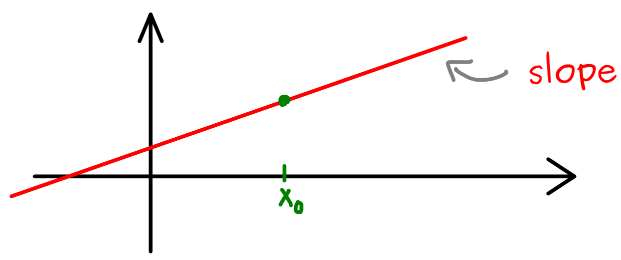
approximate f locally with a linear function?

(affine) linear function:

(linear polynomial)

$g : \mathbb{R} \to \mathbb{R}\,, \quad g(x) = a_1 \cdot x + a_0 = m \cdot (x - x_0) + \underbrace{c}_{\text{constant}}$

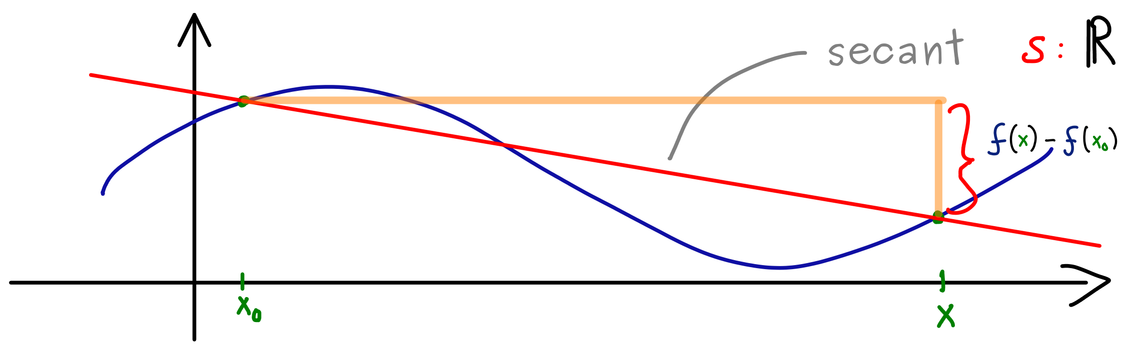
constant
$c = g(x_0)$



$$\Rightarrow \quad m = \frac{g(x) - g(x_0)}{x - x_0}\,, \quad x \neq x_0$$

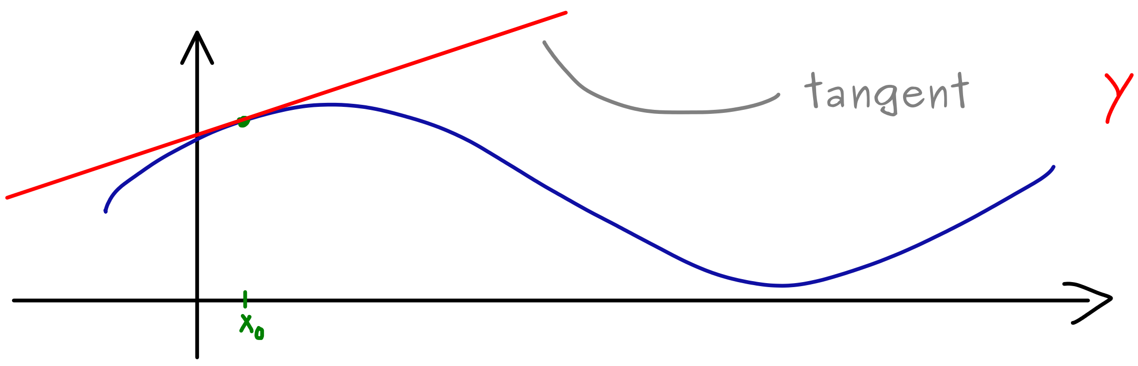
Linear approximation:

$f : \mathbb{R} \to \mathbb{R}\,, \quad x_0 \in \mathbb{R}$



$s : \mathbb{R} \to \mathbb{R}\,, \quad s(t) = m \cdot (t - x_0) + c$

$$s(t) = \frac{f(x) - f(x_0)}{x - x_0}(t - x_0) + f(x_0)$$



$y : \mathbb{R} \to \mathbb{R}$

$$y(t) = \underbrace{\lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}}_{\text{we want it to exist}}(t - x_0) + f(x_0)$$

slope at x_0 :
$$f'(x_0) := \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} =: \frac{df}{dx}(x_0)$$
differential quotient/ derivative

Definition: $I \subseteq \mathbb{R}$ interval with more than one point

or $I \subseteq \mathbb{R}$ open set, $f: I \longrightarrow \mathbb{R}$, $x_0 \in I$.

We call f differentiable at x_0 if there is a function $\Delta_{f,x_0}: I \longrightarrow \mathbb{R}$

with $f(x) = f(x_0) + (x - x_0) \cdot \Delta_{f,x_0}(x)$ for all $x \in I$

and Δ_{f,x_0} is continuous at x_0.