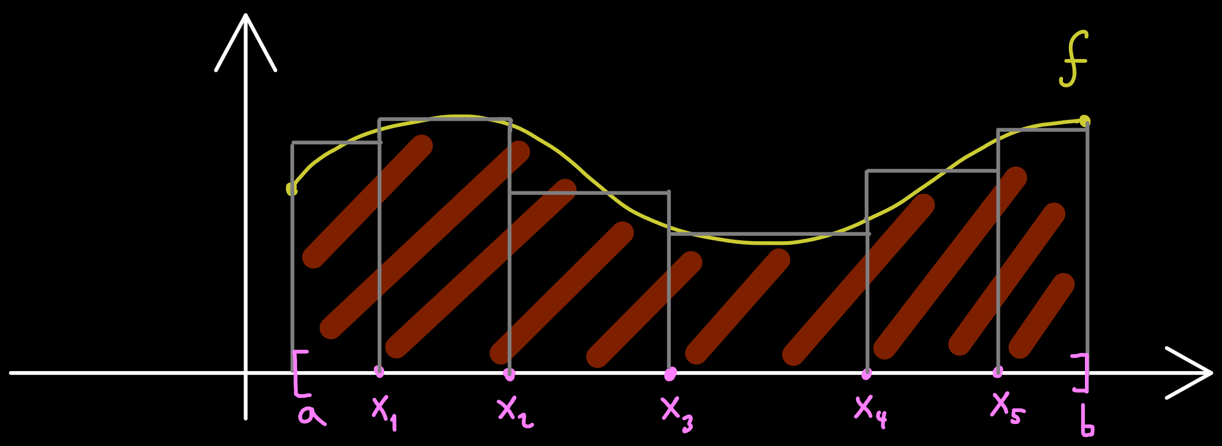
Real Analysis - Part 48



(orientated) area between graph and x-axis

$$\sum_{j=1}^{n} f(\xi_j)\cdot(x_j - x_{j-1})$$

$$\Big\downarrow n \to \infty$$

$$\int_a^b f(x)\,dx$$

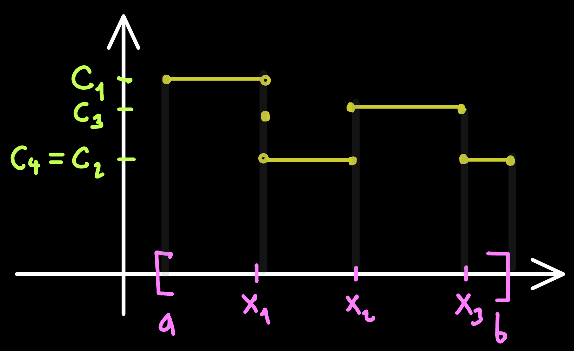
partition of x-axis $\rightsquigarrow$ Riemann integral

(more modern: Lebesgue integral)

Definition: partition of $[a,b]$: a set $\{x_0, x_1, \dots, x_n\}$ with:

$$a = x_0 < x_1 < x_2 < \cdots < x_{n-1} < x_n = b$$

Definition: $\phi : [a,b] \longrightarrow \mathbb{R}$ is called a step function if it is piecewisely constant:



there is a partition of $[a,b]$, $\{x_0, x_1, \dots, x_n\}$, and there are numbers $c_1, \dots, c_n \in \mathbb{R}$ such that

$$\phi|_{(x_{j-1},\, x_j)} = c_j \qquad \text{for all } j \in \{1, \dots, n\}$$

Can we define: $\displaystyle\int_a^b \phi(x)\,dx := \sum_{j=1}^{n} c_j \cdot (x_j - x_{j-1})$?