Real Analysis — Part 51
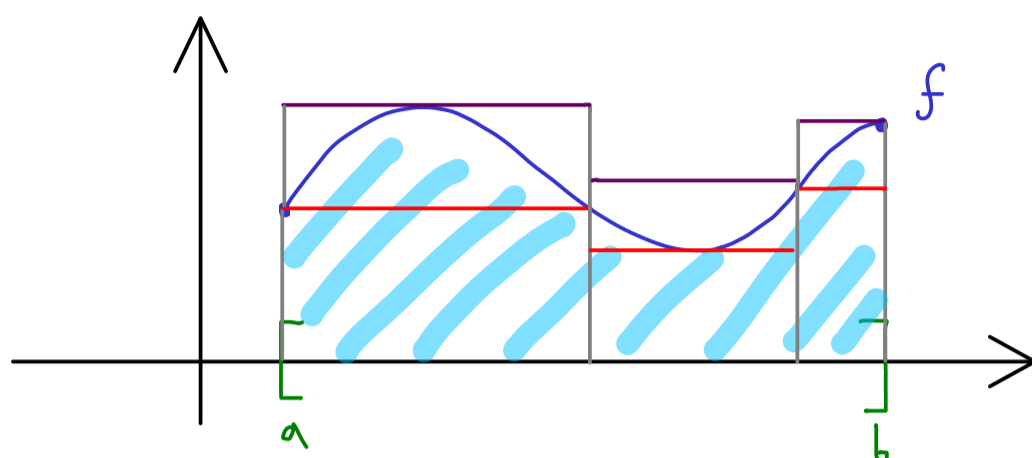

$$f: [a, b] \longrightarrow \mathbb{R}$$

bounded

Use step functions $\phi \in S([a, b])$:

$$\sup \left\{ \int_a^b \phi(x)\, dx \;\middle|\; \phi \in S([a, b]),\ \phi \leq f \right\}$$

$$\inf \left\{ \int_a^b \phi(x)\, dx \;\middle|\; \phi \in S([a, b]),\ \phi \geq f \right\}$$

<u>Definition:</u> A bounded function $f: [a, b] \longrightarrow \mathbb{R}$ is called <u>Riemann-integrable</u> if

$$\sup \left\{ \int_a^b \phi(x)\, dx \;\middle|\; \phi \in S([a, b]),\ \phi \leq f \right\} = \inf \left\{ \int_a^b \phi(x)\, dx \;\middle|\; \phi \in S([a, b]),\ \phi \geq f \right\}$$

In this case: $\displaystyle\int_a^b f(x)\, dx$ is called the (Riemann) <u>integral of</u> f