BECOME A MEMBER

ON STEADY

The Bright Side of Mathematics

Real Analysis – Part 2

Sequences: A sequence of real numbers:

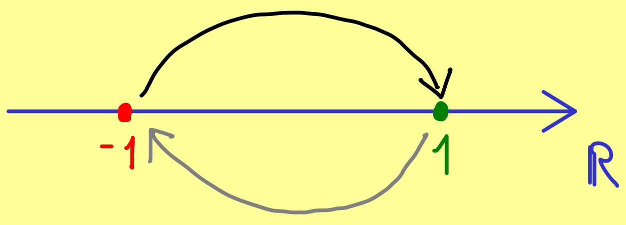
a map $a: \mathbb{N} \longrightarrow \mathbb{R}$

or $a: \mathbb{N}_0 \longrightarrow \mathbb{R}$

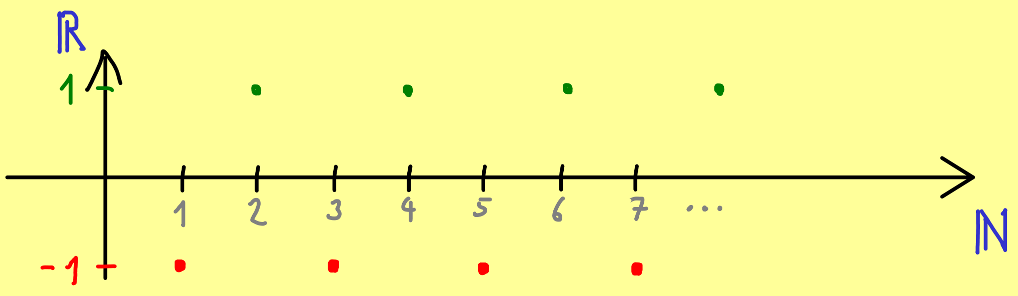
Notations: $(a_1, a_2, a_3, \dots)$ infinite list of numbers

$(a_n)_{n\in\mathbb{N}}$ or $(a_n)_{n=1}^{\infty}$ or (a_n)

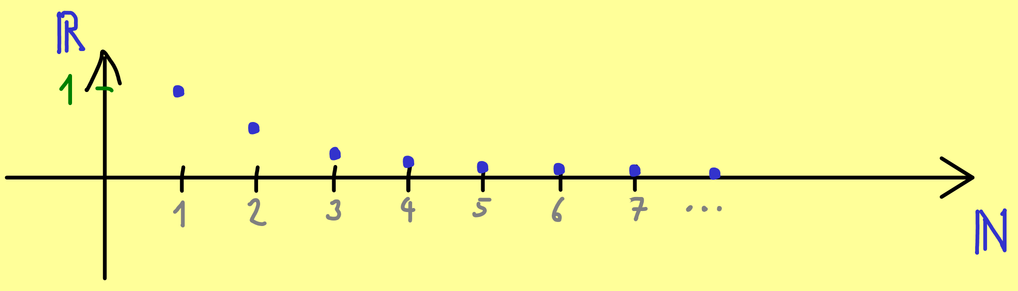
Examples: (a) $(a_n)_{n\in\mathbb{N}} = \left((-1)^n\right)_{n\in\mathbb{N}} = (-1, 1, -1, 1, \dots)$



Graph:



(b) $(a_n)_{n\in\mathbb{N}} = \left(\frac{1}{n}\right)_{n\in\mathbb{N}} = \left(1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}, \dots\right)$

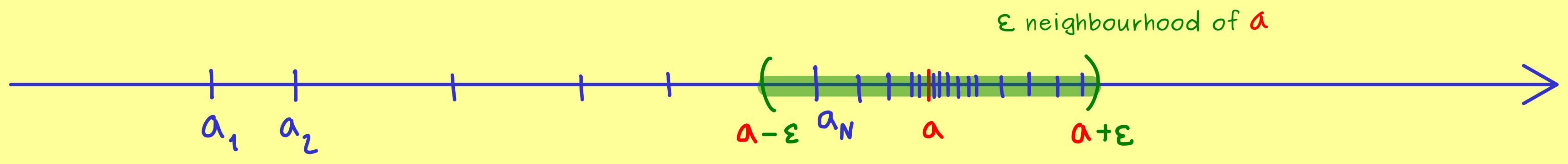


We will see: $\lim_{n\to\infty} a_n = 0$

(c) $(a_n)_{n\in\mathbb{N}} = \left(2^n\right)_{n\in\mathbb{N}} = (2, 4, 8, 16, 32, 64, 128, 256, \dots)$

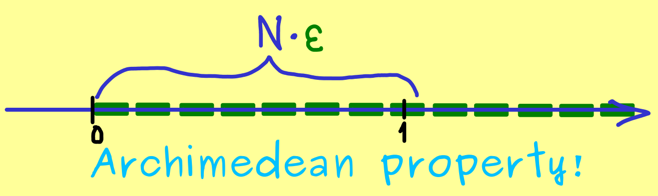
Definition: A sequence $(a_n)_{n\in\mathbb{N}}$ is called convergent to $a \in \mathbb{R}$ if

$$\forall \varepsilon > 0 \quad \exists N \in \mathbb{N} \quad \forall n \geq N : \quad |a_n - a| < \varepsilon$$



If there is no such $a \in \mathbb{R}$, we call the sequence $(a_n)_{n\in\mathbb{N}}$ divergent.

Example: $(a_n)_{n\in\mathbb{N}} = \left(\frac{1}{n}\right)_{n\in\mathbb{N}}$ is convergent to $0 \in \mathbb{R}$.



Proof: Let $\varepsilon > 0$. We choose $N \in \mathbb{N}$ such that $N \cdot \varepsilon > 1$.

Then for $n \geq N$, we have: $|a_n - 0| = |a_n| = \frac{1}{n} \leq \frac{1}{N} < \varepsilon$.