The Bright Side of Mathematics

The following pages cover the whole Start Learning Logic course of the Bright Side of Mathematics. Please note that the creator lives from generous supporters and would be very happy about a donation. See more here: https://tbsom.de/support

Have fun learning mathematics!

Start Learning Logic – Part 1

Logical statement (proposition): statement that is either True or False

Examples:

(a) Mars is a planet (True logical statement)

(b) Pluto is a planet (False logical statement)

(c) $1 + 1 = 2$ (True logical statement)

(d) The number 5 is smaller than the number 2 (False logical statement)

(e) Good morning! (Not a logical statement)

(f) $x + 1 = 1$ (Not a logical statement) ⇝ predicate

Logical operations:

Negation: For a logical statement A, $\neg A$ denotes the negation.

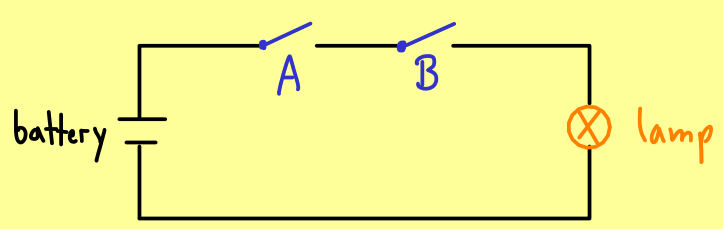
Truth table

A	$\neg A$
T	F
F	T

Examples: (a) A = The wine bottle is full
$\neg A$ = The wine bottle is not full

(b) $A = 2 + 2 = 5$
$\neg A = 2 + 2 \neq 5$

Conjunction: For two logical statements A, B, $A \wedge B$ denotes the conjuction.



Truth table

A	B	$A \wedge B$
T	T	T
T	F	F
F	T	F
F	F	F

Start Learning Logic – Part 2

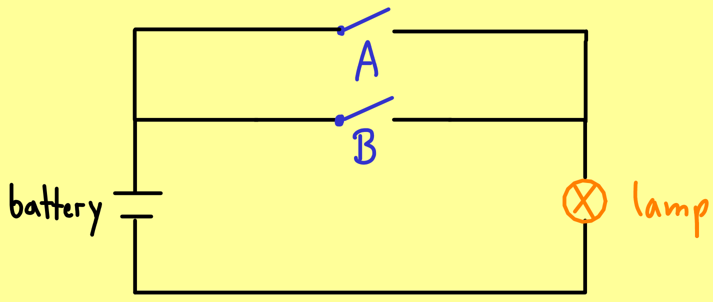
Logical statements A, B $\rightsquigarrow$ new logical statements $\neg A$, $A \wedge B$

logical variables

logical operations

Logical operations:

Disjunction: For two logical statements A, B, $A \vee B$ denotes the disjunction.



Truth table

A	B	$A \vee B$
T	T	T
T	F	T
F	T	T
F	F	F

Example: $\neg A \vee A$

Truth table

A	$\neg A$	$\neg A \vee A$
T	F	T
F	T	T

We say $\neg A \vee A$ is a tautology.

always true (independent of the truth values of the logical variables that are contained)

Logical equivalence:

Two logical statements are called logically equivalent if the truth tables (all possible assignments of truth values for the logical variables) are the same.

Example: $\neg(A \vee B) \Leftrightarrow (\neg A) \wedge (\neg B)$

A	B	$A \vee B$	$\neg A$	$\neg B$	$\neg(A \vee B)$	$\neg A \wedge \neg B$
T	T	T	F	F	F	F
T	F	T	F	T	F	F
F	T	T	T	F	F	F
F	F	F	T	T	T	T

The Bright Side of Mathematics

Start Learning Logic – Part 3

Logical operations:

Conditional: For two logical statements A, B, $A \to B$ denotes the conditional.

Truth table

A	B	$A \to B$
T	T	T
T	F	F
F	T	T
F	F	T

$\Rightarrow$ means $\to$ gives tautology

A	B	$A \wedge B$	$A \wedge B \to B$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

tautology

We can write:

$A \wedge B \Rightarrow B$

Biconditional: For two logical statements A, B, $A \leftrightarrow B$ denotes the biconditional.

$\Leftrightarrow$ means $\leftrightarrow$ gives tautology

Truth table

A	B	$A \leftrightarrow B$
T	T	T
T	F	F
F	T	F
F	F	T

Example: (a) $A \leftrightarrow B \quad \Leftrightarrow \quad (A \to B) \wedge (B \to A)$

(b) $A \to B \quad \Leftrightarrow \quad \neg B \to \neg A$ (contraposition)

If there is fog, then we have poor visibility

If we don't have poor visibility, there is no fog.

Deduction rules: (how to get new true propositions from other true propositions)

Modus ponens: If $A \to B$ true and A true, then: B true

A	B	$A \to B$
T	T	T
T	F	F
F	T	T
F	F	T

Chain syllogism: If $A \to B$ true and $B \to C$ true, then: $A \to C$ true

Reductio ad absurdum: If $A \to B$ true and $A \to \neg B$ true, then: $\neg A$ true