

**Problem 4** *Fixed points* (2 bonus points)

Let $g : [0, 1] \rightarrow [0, 1]$ be a continuous function. Prove that there exists some $c \in [0, 1]$ with $g(c) = c$. Such a c is called a *fixed point* of g .

Solutions

$h(x) := g(x) - x$ is also continuous.

1. Case: $h(0) = 0 \Rightarrow 0$ is a fixed point of g

2. Case: $h(1) = 0 \Rightarrow 1$ is a fixed point of g

3. Case: $h(0) > 0$ ($h(0) < 0 \Rightarrow g(0) < 0$ not possible!)

and $h(1) < 0$ ($h(1) > 0 \Rightarrow g(1) > 1$ not possible!)

\Rightarrow Intermediate theorem says there is a $c \in \mathbb{R}$ with

$h(c) = 0 \Rightarrow g(c) = c \Rightarrow c$ fixed point.