Problem 3 *Antiderivatives* (4 bonus points)
Let $n \in \mathbb{N}$ and $a > 0$. Calculate the antiderivatives of the following functions:

a) $6x^4 - 8x^3 + \frac{23}{5}x^2 - 34x + 502$

b) $x^n \log(x)$

c) $\frac{1}{x \log(x)}$

d) $\frac{1}{x\sqrt{a^2 + x^2}}$

e) $\cos^2(x)\sin(x)$

f) $\sqrt{\frac{x+1}{x-1}}$

g) $\frac{\sin(2x)}{1+\cos^2(x)}$

h) $\frac{x^7}{x^4+1}$

Solutions

(a) $\frac{6}{5}x^5 - 2x^4 + \frac{23}{15}x^3 - 17x^2 + 502x + C$

(b) $\frac{x^{n+1}}{n+1}\log(x) - \frac{x^{n+1}}{(n+1)^2} + C$

(c) $\log|\log(x)| + C$

(d) $-\frac{1}{a}\operatorname{arsinh}\left(\frac{a}{x}\right) + C$

(e) $-\frac{1}{3}\cos^3(x) + C$

(f) $\sqrt{x^2-1} + \operatorname{arcosh}(x) + C$

(g) $-\log[1 + \cos^2(x)] + C$

(h) $\frac{1}{4}(x^4 + 1 - \log(x^4+1)) + C$