BECOME A MEMBER

ON STEADY

The Bright Side of Mathematics

Problem 1:

Show that the function f given by

$$f(x) = \begin{cases} x, & x \in \mathbb{Q} \\ 0, & x \notin \mathbb{Q} \end{cases}$$

is continuous at $x = 0$

Problem 2:

For which values a, b is the following function continuous?

$f: \mathbb{R} \to \mathbb{R}$

$$f(x) := \begin{cases} x^4 - 4 & \text{if } x \leq -2 \\ ax + b & \text{if } -2 < x < 4 \\ \sqrt{x} & \text{if } 4 \leq x \end{cases}$$

Problem 3:

Let $I = [0,1]$ and $f: I \longrightarrow I$ be continuous.

(a) Show that f has a fixed point $\tilde{x}$ $\left(f(\tilde{x}) = \tilde{x} \right)$

(b) Show that for $f: \mathbb{R} \longrightarrow \mathbb{R}$ the statement in (a) is not correct. (in general)

Problem 4:

$f: [0,1] \longrightarrow \mathbb{R}$ given by $f(x) = x(1-x)$

Show that f is continuous with the ε-δ-definition.

Problem 5:

Let $f: \mathbb{R} \to \mathbb{R}$ be continuous in x_0.

Show by using the ε-δ-definition that, if $f(x_0) > 0$, there exists $\delta > 0$ such that $f(x) > 0$ for all x in $(x_0 - \delta, x_0 + \delta)$.

Problem 6:

$f_n(x) = \dfrac{nx}{1 + |nx|}$ continuous? What about the pointwise limit?

Problem 7:

Let $f: \mathbb{R} \to \mathbb{R}$ be Lipschitz continuous.

Show by using the ε-δ-definition that f is also continuous on $\mathbb{R}$.